These power lines transfer energy from the power company to homes and businesses. The energy is transferred at a very high voltage, possibly hundreds of thousands of volts in some cases. Despite the fact that this makes power lines very dangerous, the high voltage results in less loss of power due to resistance in the wires. (Telegraph Colour Library/FPG)
Thus far our treatment of electrical phenomena has been confined to the study of charges in equilibrium situations, or electrostatics. We now consider situations involving electric charges that are not in equilibrium. We use the term electric current, or simply current, to describe the rate of flow of charge through some region of space. Most practical applications of electricity deal with electric currents. For example, the battery in a flashlight produces a current in the filament of the bulb when the switch is turned on. A variety of home appliances operate on alternating current. In these common situations, current exists in a conductor, such as a copper wire. It also is possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

This chapter begins with the definition of current. A microscopic description of current is given, and some of the factors that contribute to the opposition to the flow of charge in conductors are discussed. A classical model is used to describe electrical conduction in metals, and some of the limitations of this model are cited. We also define electrical resistance and introduce a new circuit element, the resistor. We conclude by discussing the rate at which energy is transferred to a device in an electric circuit.

## 27.1 Electric Current

In this section, we study the flow of electric charges through a piece of material. The amount of flow depends on the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric current is said to exist.

It is instructive to draw an analogy between water flow and current. In many localities it is common practice to install low-flow showerheads in homes as a water-conservation measure. We quantify the flow of water from these and similar devices by specifying the amount of water that emerges during a given time interval, which is often measured in liters per minute. On a grander scale, we can characterize a river current by describing the rate at which the water flows past a particular location. For example, the flow over the brink at Niagara Falls is maintained at rates between 1,400 m$^3$/s and 2,800 m$^3$/s.

There is also an analogy between thermal conduction and current. In Section 20.7, we discussed the flow of energy by heat through a sample of material. The rate of energy flow is determined by the material as well as the temperature difference across the material, as described by Equation 20.14.

To define current more precisely, suppose that charges are moving perpendicular to a surface of area $A$, as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.) The **current is the rate at which charge flows through this surface.** If $\Delta Q$ is the amount of charge that passes through this area in a time interval $\Delta t$, the **average current $I_{av}$** is equal to the charge that passes through $A$ per unit time:

$$ I_{av} = \frac{\Delta Q}{\Delta t} \tag{27.1} $$

![Figure 27.1 Charges in motion through an area $A$. The time rate at which charge flows through the area is defined as the current $I$. The direction of the current is the direction in which positive charges flow when free to do so.](image)
If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** \( I \) as the differential limit of average current:

\[
I = \frac{dQ}{dt}
\]  

(27.2) **Electric current**

The SI unit of current is the **ampere** (A):

\[
1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}}
\]  

(27.3)

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

The charges passing through the surface in Figure 27.1 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, the **direction of the current is opposite the direction of flow of electrons.** However, if we are considering a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance—the current is the result of the flow of both positive and negative charges.

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential, and hence the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire, and therefore there is no current. However, if the ends of the conducting wire are connected to a battery, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the conduction electrons in the wire, causing them to move in the wire, thus creating a current.

It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier**. For example, the mobile charge carriers in a metal are electrons.

**Microscopic Model of Current**

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a conductor of cross-sectional area \( A \) (Fig. 27.2). The volume of a section of the conductor of length \( \Delta x \) (the gray region shown in Fig. 27.2) is \( A \Delta x \). If \( n \) represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is \( nA \Delta x \). Therefore, the total charge \( \Delta Q \) in this section is

\[
\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA \Delta x)q
\]

where \( q \) is the charge on each carrier. If the carriers move with a speed \( v_d \), the displacement they experience in the \( x \) direction in a time interval \( \Delta t \) is \( \Delta x = v_d \Delta t \). Let us choose \( \Delta t \) to be the time interval required for the charges in the cylinder to move through a displacement whose magnitude is equal to the length of the cylinder. This time interval is also that required for all of the charges in the cylinder to pass through the circular area at one end. With this choice, we can write \( \Delta Q \) in the form

\[
\Delta Q = (nA v_d \Delta t)q
\]

If we divide both sides of this equation by \( \Delta t \), we see that the average current in the conductor is

\[
I_{av} = \frac{\Delta Q}{\Delta t} = nqv_d A
\]  

(27.4) **Current in a conductor in terms of microscopic quantities**
The speed of the charge carriers \( v_d \) is an average speed called the **drift speed**. To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated—that is, the potential difference across it is zero—then these electrons undergo random motion that is analogous to the motion of gas molecules. As we discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. However, the electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag (Fig. 27.3). Despite the collisions, the electrons move slowly along the conductor (in a direction opposite that of \( \mathbf{E} \)) at the drift velocity \( v_d \).

We can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collisions causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

---

**Quick Quiz 27.1** Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions, from lowest to highest.

![Figure 27.4](image)

**Quick Quiz 27.2** Electric charge is conserved. As a consequence, when current arrives at a junction of wires, the charges can take either of two paths out of the junction and the numerical sum of the currents in the two paths equals the current that entered the junction. Thus, current is (a) a vector (b) a scalar (c) neither a vector nor a scalar.

---

**Example 27.1 Drift Speed in a Copper Wire**

The 12-gauge copper wire in a typical residential building has a cross-sectional area of \( 3.31 \times 10^{-6} \text{ m}^2 \). If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm\(^3\).

**Solution** From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro’s number of atoms \( (6.02 \times 10^{23}) \). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

\[
V = \frac{m}{\rho} = \frac{63.5 \text{ g}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3
\]

Because each copper atom contributes one free electron to the current, we have:

\[
n = \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3} \left( \frac{1.00 \times 10^6 \text{ cm}^3}{1 \text{ m}^3} \right)
= 8.49 \times 10^{28} \text{ electrons/m}^3
\]

From Equation 27.4, we find that the drift speed is

\[
v_d = \frac{I}{nqA}
\]

where \( q \) is the absolute value of the charge on each electron. Thus,

\[
v_d = \frac{I}{nqA}
= \frac{10.0 \text{ C/s}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(3.31 \times 10^{-6} \text{ m}^2)}
= 2.22 \times 10^{-4} \text{ m/s}
\]
Example 27.1 shows that typical drift speeds are very low. For instance, electrons traveling with a speed of $2.22 \times 10^{-4} \text{ m/s}$ would take about 75 min to travel 1 m! In view of this, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, changes in the electric field that drives the free electrons travel through the conductor with a speed close to that of light. Thus, when you flip on a light switch, electrons already in the filament of the lightbulb experience electric forces and begin moving after a time interval on the order of nanoseconds.

### 27.2 Resistance

In Chapter 24 we found that the electric field inside a conductor is zero. However, this statement is true only if the conductor is in static equilibrium. The purpose of this section is to describe what happens when the charges in the conductor are not in equilibrium, in which case there is an electric field in the conductor.

Consider a conductor of cross-sectional area $A$ carrying a current $I$. The current density $J$ in the conductor is defined as the current per unit area. Because the current $I = nqv_d A$, the current density is

$$J = \frac{I}{A} = nqv_d \quad (27.5)$$

where $J$ has SI units of $\text{A/m}^2$. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area $A$ is perpendicular to the direction of the current. In general, current density is a vector quantity:

$$\mathbf{J} = nq\mathbf{v}_d \quad (27.6)$$

From this equation, we see that current density is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

A current density $\mathbf{J}$ and an electric field $\mathbf{E}$ are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E} \quad (27.7)$$

where the constant of proportionality $\sigma$ is called the conductivity of the conductor.\(^1\) Materials that obey Equation 27.7 are said to follow Ohm’s law, named after Georg Simon Ohm (1789–1854). More specifically, Ohm’s law states that

for many materials (including most metals), the ratio of the current density to the electric field is a constant $\sigma$ that is independent of the electric field producing the current.

Materials that obey Ohm’s law and hence demonstrate this simple relationship between $\mathbf{E}$ and $\mathbf{J}$ are said to be ohmic. Experimentally, however, it is found that not all materials have this property. Materials and devices that do not obey Ohm’s law are said to be nonohmic. Ohm’s law is not a fundamental law of nature but rather an empirical relationship valid only for certain materials.

We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area $A$ and length $L$, as shown in

\(^1\) Do not confuse conductivity $\sigma$ with surface charge density, for which the same symbol is used.
In Chapter 5, we introduced Newton’s second law, \( \sum F = ma \), for a net force on an object of mass \( m \). This can be written as

\[ m = \frac{\sum F}{a} \]

In that chapter, we defined mass as resistance to a change in motion in response to an external force. Mass as resistance to changes in motion is analogous to electrical resistance to charge flow, and Equation 27.8 is analogous to the form of Newton’s second law shown here.

## PITFALL PREVENTION

### 27.3 We’ve Seen Something Like Equation 27.8 Before

In Chapter 5, we introduced Newton’s second law, \( \sum F = ma \), for a net force on an object of mass \( m \). This can be written as

\[ m = \frac{\sum F}{a} \]

In that chapter, we defined mass as resistance to a change in motion in response to an external force. Mass as resistance to changes in motion is analogous to electrical resistance to charge flow, and Equation 27.8 is analogous to the form of Newton’s second law shown here.

## PITFALL PREVENTION

### 27.4 Equation 27.8 Is Not Ohm’s Law

Many individuals call Equation 27.8 Ohm’s law, but this is incorrect. This equation is simply the definition of resistance, and provides an important relationship between voltage, current, and resistance. Ohm's law is related to a linear relationship between \( J \) and \( E \) (Eq. 27.7) or, equivalently, between \( I \) and \( \Delta V \), which, from Equation 27.8, indicates that the resistance is constant, independent of the applied voltage.

Figure 27.5. A potential difference \( \Delta V = V_a - V_b \) is maintained across the conductor, setting up an electric field \( E \), and this field produces a current \( I \) that is proportional to the potential difference.

\[ \Delta V = E\ell \]

Therefore, we can express the magnitude of the current density in the wire as

\[ J = \sigma E = \sigma \frac{\Delta V}{\ell} \]

Because \( J = I/A \), we can write the potential difference as

\[ \Delta V = \frac{\ell}{\sigma} J = \left( \frac{\ell}{\sigma A} \right) I = RI \]

The quantity \( R = \ell/\sigma A \) is called the **resistance** of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

\[ R = \frac{\Delta V}{I} \tag{27.8} \]

We will use this equation over and over again when studying electric circuits. From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be one **ohm** (\( \Omega \)):

\[ 1 \Omega = \frac{1 V}{1 A} \tag{27.9} \]

This expression shows that if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 \( \Omega \). For example, if an electrical appliance connected to a 120-V source of potential difference carries a current of 6 A, its resistance is 20 \( \Omega \).

The inverse of conductivity is **resistivity** \( \rho \):

\[ \rho = \frac{1}{\sigma} \tag{27.10} \]

where \( \rho \) has the units ohm-meters (\( \Omega \cdot m \)). Because \( R = \ell/\sigma A \), we can express the resistance of a uniform block of material along the length \( \ell \) as

\[ R = \rho \frac{\ell}{A} \tag{27.11} \]

\[ ^2 \text{This result follows from the definition of potential difference:} \]

\[ V_b - V_a = - \int_a^b \mathbf{E} \cdot d\mathbf{s} = E \int_a^b dx = E\ell \]

\[ ^3 \text{Do not confuse resistivity} \rho \text{ with mass density or charge density, for which the same symbol is used.} \]
Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. Additionally, as you can see from Equation 27.11, the resistance of a sample depends on geometry as well as on resistivity. Table 27.1 gives the resistivities of a variety of materials at 20°C. Note the enormous range, from very low values for good conductors such as copper and silver, to very high values for good insulators such as glass and rubber. An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Equation 27.11 shows that the resistance of a given cylindrical conductor such as a wire is proportional to its length and inversely proportional to its cross-sectional area. If the length of a wire is doubled, then its resistance doubles. If its cross-sectional area is doubled, then its resistance decreases by one half. The situation is analogous to the flow of a liquid through a pipe. As the pipe’s length is increased, the resistance to flow increases. As the pipe’s cross-sectional area is increased, more liquid crosses a given cross section of the pipe per unit time interval. Thus, more liquid flows for the same pressure differential applied to the pipe, and the resistance to flow decreases.

### Table 27.1

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity(\Omega \cdot \text{m})</th>
<th>Temperature Coefficient(\alpha(\text{°C})^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>(1.59 \times 10^{-8})</td>
<td>(3.8 \times 10^{-3})</td>
</tr>
<tr>
<td>Copper</td>
<td>(1.7 \times 10^{-8})</td>
<td>(3.9 \times 10^{-3})</td>
</tr>
<tr>
<td>Gold</td>
<td>(2.44 \times 10^{-8})</td>
<td>(3.4 \times 10^{-3})</td>
</tr>
<tr>
<td>Aluminum</td>
<td>(2.82 \times 10^{-8})</td>
<td>(3.9 \times 10^{-3})</td>
</tr>
<tr>
<td>Tungsten</td>
<td>(5.6 \times 10^{-8})</td>
<td>(4.5 \times 10^{-3})</td>
</tr>
<tr>
<td>Iron</td>
<td>(10 \times 10^{-8})</td>
<td>(5.0 \times 10^{-3})</td>
</tr>
<tr>
<td>Platinum</td>
<td>(11 \times 10^{-8})</td>
<td>(3.92 \times 10^{-3})</td>
</tr>
<tr>
<td>Lead</td>
<td>(22 \times 10^{-8})</td>
<td>(3.9 \times 10^{-3})</td>
</tr>
<tr>
<td>Nichrome(^c)</td>
<td>(1.50 \times 10^{-6})</td>
<td>(0.4 \times 10^{-3})</td>
</tr>
<tr>
<td>Carbon</td>
<td>(3.5 \times 10^{-5})</td>
<td>(-0.5 \times 10^{-3})</td>
</tr>
<tr>
<td>Germanium</td>
<td>(0.46)</td>
<td>(-48 \times 10^{-3})</td>
</tr>
<tr>
<td>Silicon</td>
<td>(640)</td>
<td>(-75 \times 10^{-3})</td>
</tr>
<tr>
<td>Glass</td>
<td>(10^{10}) to (10^{14})</td>
<td></td>
</tr>
<tr>
<td>Hard rubber</td>
<td>(~10^{13})</td>
<td></td>
</tr>
<tr>
<td>Sulfur</td>
<td>(10^{13})</td>
<td></td>
</tr>
<tr>
<td>Quartz (fused)</td>
<td>(75 \times 10^{16})</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) All values at 20°C.

\(^b\) See Section 27.4.

\(^c\) A nickel–chromium alloy commonly used in heating elements.

### PITFALL PREVENTION

#### 27.5 Resistance and Resistivity

Resistivity is property of a substance, while resistance is a property of an object. We have seen similar pairs of variables before. For example, density is a property of a substance, while mass is a property of an object. Equation 27.11 relates resistance to resistivity, and we have seen a previous equation (Equation 1.1) which relates mass to density.

![An assortment of resistors used in electrical circuits.](image-url)
Most electric circuits use circuit elements called **resistors** to control the current level in the various parts of the circuit. Two common types of resistors are the composition resistor, which contains carbon, and the wire-wound resistor, which consists of a coil of wire. Values of resistors in ohms are normally indicated by color-coding, as shown in Figure 27.6 and Table 27.2.

Ohioan materials and devices have a linear current–potential difference relationship over a broad range of applied potential differences (Fig. 27.7a). The slope of the $I$-versus-$\Delta V$ curve in the linear region yields a value for $1/R$. Nonohmic materials have a nonlinear current–potential difference relationship. One common semiconducting device that has nonlinear $I$-versus-$\Delta V$ characteristics is the junction diode (Fig. 27.7b). The resistance of this device is low for currents in one direction (positive $\Delta V$) and high for currents in the reverse direction (negative $\Delta V$). In fact, most modern electronic devices, such as transistors, have nonlinear current–potential difference relationships; their proper operation depends on the particular way in which they violate Ohm’s law.

**Quick Quiz 27.3** Suppose that a current-carrying ohmic metal wire has a cross-sectional area that gradually becomes smaller from one end of the wire to the other. The current must have the same value in each section of the wire so that charge does not accumulate at any one point. How do the drift velocity and the resistance per

![Figure 27.6](image-url)

**Figure 27.6** The colored bands on a resistor represent a code for determining resistance. The first two colors give the first two digits in the resistance value. The third color represents the power of ten for the multiplier of the resistance value. The last color is the tolerance of the resistance value. As an example, the four colors on the circled resistors are red (2), black (0), orange (10^3), and gold (5%), and so the resistance value is $2 \times 10^3 \, \Omega = 20 \, k\Omega$ with a tolerance value of 5% = 1 kΩ. (The values for the colors are from Table 27.2.)

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
<th>Multiplier</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>$10^1$</td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>$10^2$</td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>$10^3$</td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>$10^4$</td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>$10^5$</td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>$10^6$</td>
<td></td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>$10^7$</td>
<td></td>
</tr>
<tr>
<td>Gray</td>
<td>8</td>
<td>$10^8$</td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>$10^9$</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>$10^{-1}$</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>$10^{-2}$</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Colorless</td>
<td></td>
<td>20%</td>
<td></td>
</tr>
</tbody>
</table>

**Table 27.2**

**Color Coding for Resistors**

![Figure 27.7](image-url)

**Figure 27.7** (a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a junction diode. This device does not obey Ohm’s law.
unit length vary along the wire as the area becomes smaller? (a) The drift velocity and resistance both increase. (b) The drift velocity and resistance both decrease. (c) The drift velocity increases and the resistance decreases. (d) The drift velocity decreases and the resistance increases.

**Quick Quiz 27.4** A cylindrical wire has a radius $r$ and length $\ell$. If both $r$ and $\ell$ are doubled, the resistance of the wire (a) increases (b) decreases (c) remains the same.

**Quick Quiz 27.5** In Figure 27.7b, as the applied voltage increases, the resistance of the diode (a) increases (b) decreases (c) remains the same.

---

**Example 27.2  The Resistance of a Conductor**

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of $2.00 \times 10^{-4}$ m$^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}$ $\Omega \cdot$ m.

**Solution** From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8} \Omega \cdot m) \left( \frac{0.100 \text{ m}}{2.00 \times 10^{-4} \text{ m}^2} \right) = 1.41 \times 10^{-5} \Omega$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10} \Omega \cdot m) \left( \frac{0.100 \text{ m}}{2.00 \times 10^{-4} \text{ m}^2} \right) = 1.5 \times 10^{13} \Omega$$

As you might guess from the large difference in resistivities, the resistances of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

---

**Example 27.3  The Resistance of Nichrome Wire**

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

**Solution** The cross-sectional area of this wire is

$$A = \pi r^2 = \pi (0.321 \times 10^{-3} \text{ m})^2 = 3.24 \times 10^{-7} \text{ m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}$ $\Omega \cdot$ m (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6} \Omega \cdot \text{m}}{3.24 \times 10^{-7} \text{ m}^2} = 4.6 \Omega/\text{m}$$

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

**Solution** Because a 1.0-m length of this wire has a resistance of 4.6 $\Omega$, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10 \text{ V}}{4.6 \Omega} = 2.2 \text{ A}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only $0.052 \Omega/\text{m}$. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

---

**Example 27.4  The Radial Resistance of a Coaxial Cable**

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with silicon, as shown in Figure 27.8a, and current leakage through the silicon, in the radial direction, is unwanted. (The cable is designed to conduct current along its length—this is not the current we are considering here.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer one is $b = 1.75$ cm, and the length is $L = 15.0$ cm.
Calculate the resistance of the silicon between the two conductors.

**Solution** Conceptualize by imagining two currents, as suggested in the text of the problem. The desired current is along the cable, carried within the conductors. The undesired current corresponds to charge leakage through the silicon and its direction is radial. Because we know the resistivity and the geometry of the silicon, we categorize this as a problem in which we find the resistance of the silicon from these parameters, using Equation 27.11. Because the area through which the charges pass depends on the radial position, we must use integral calculus to determine the answer.

To analyze the problem, we divide the silicon into concentric elements of infinitesimal thickness $dr$ (Fig. 27.8b). We start by using the differential form of Equation 27.11, replacing $\ell$ with $r$ for the distance variable: $dR = \rho dr/A$, where $dR$ is the resistance of an element of silicon of thickness $dr$ and surface area $A$. In this example, we take as our representative concentric element a hollow silicon cylinder of radius $r$, thickness $dr$, and length $L$, as in Figure 27.8. Any charge that passes from the inner conductor to the outer one must pass radially through this concentric element, and the area through which this charge passes is $A = 2\pi r L$. (This is the curved surface area—circumference multiplied by length—of our hollow silicon cylinder of thickness $dr$.) Hence, we can write the resistance of our hollow cylinder of silicon as

$$dR = \frac{\rho}{2\pi r L} dr$$

Because we wish to know the total resistance across the entire thickness of the silicon, we must integrate this expression from $r = a$ to $r = b$:

$$R = \int_a^b dR = \int_a^b \frac{\rho}{2\pi r L} dr = \frac{\rho}{2\pi L} \ln \left( \frac{b}{a} \right)$$

Substituting in the values given, and using $\rho = 640 \, \Omega \cdot \text{m}$ for silicon, we obtain

$$R = \frac{640 \, \Omega \cdot \text{m}}{2\pi (0.150 \, \text{m})} \ln \left( \frac{1.75 \, \text{cm}}{0.500 \, \text{cm}} \right) = 851 \Omega$$

To finalize this problem, let us compare this resistance to that of the inner conductor of the cable along the 15.0-cm length. Assuming that the conductor is made of copper, we have

$$R = \rho \frac{\ell}{A} = (1.7 \times 10^{-8} \, \Omega \cdot \text{m}) \left( \frac{0.150 \, \text{m}}{\pi (5.00 \times 10^{-3} \, \text{m})^2} \right) = 3.2 \times 10^{-5} \Omega$$

This resistance is much smaller than the radial resistance. As a consequence, almost all of the current corresponds to charge moving along the length of the cable, with a very small fraction leaking in the radial direction.

**What If?** Suppose the coaxial cable is enlarged to twice the overall diameter with two possibilities: (1) the ratio $b/a$ is held fixed, or (2) the difference $b - a$ is held fixed. For which possibility does the leakage current between the inner and outer conductors increase when the voltage is applied between the two conductors?

**Answer** In order for the current to increase, the resistance must decrease. For possibility (1), in which $b/a$ is held fixed, Equation (1) tells us that the resistance is unaffected. For possibility (2), we do not have an equation involving the difference $b - a$ to inspect. Looking at Figure 27.8b, however, we see that increasing $b$ and $a$ while holding the voltage constant results in charge flowing through the same thickness of silicon but through a larger overall area perpendicular to the flow. This larger area will result in lower resistance and a higher current.
27.3 A Model for Electrical Conduction

In this section we describe a classical model of electrical conduction in metals that was first proposed by Paul Drude (1863–1906) in 1900. This model leads to Ohm’s law and shows that resistivity can be related to the motion of electrons in metals. Although the Drude model described here does have limitations, it nevertheless introduces concepts that are still applied in more elaborate treatments.

Consider a conductor as a regular array of atoms plus a collection of free electrons, which are sometimes called conduction electrons. The conduction electrons, although bound to their respective atoms when the atoms are not part of a solid, gain mobility when the free atoms condense into a solid. In the absence of an electric field, the conduction electrons move in random directions through the conductor with average speeds on the order of $10^6$ m/s. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an electron gas.

There is no current in the conductor in the absence of an electric field because the drift velocity of the free electrons is zero. That is, on the average, just as many electrons move in one direction as in the opposite direction, and so there is no net flow of charge.

This situation changes when an electric field is applied. Now, in addition to undergoing the random motion just described, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed $v_d$ that is much smaller (typically $10^{-4}$ m/s) than their average speed between collisions (typically $10^6$ m/s).

Figure 27.9 provides a crude description of the motion of free electrons in a conductor. In the absence of an electric field, there is no net displacement after many collisions (Fig. 27.9a). An electric field $E$ modifies the random motion and causes the electrons to drift in a direction opposite that of $E$ (Fig. 27.9b).

In our model, we assume that the motion of an electron after a collision is independent of its motion before the collision. We also assume that the excess energy acquired by the electrons in the electric field is lost to the atoms of the conductor when the electrons and atoms collide. The energy given up to the atoms increases their vibrational energy, and this causes the temperature of the conductor to increase. The temperature increase of a conductor due to resistance is utilized in electric toasters and other familiar appliances.

We are now in a position to derive an expression for the drift velocity. When a free electron of mass $m_e$ and charge $q (= e)$ is subjected to an electric field $E$, it experiences a force $F = qE$. Because this force is related to the acceleration of the electron through Newton’s second law, $F = m_e a$, we conclude that the acceleration of the electron is

$$a = \frac{qE}{m_e} \quad (27.12)$$
This acceleration, which occurs for only a short time interval between collisions, enables the electron to acquire a small drift velocity. If \( \mathbf{v}_i \) is the electron’s initial velocity the instant after a collision (which occurs at a time that we define as \( t = 0 \)), then the velocity of the electron at time \( t \) (at which the next collision occurs) is

\[
\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \mathbf{v}_i + \frac{qE}{m_e} t
\]  

(27.13)

We now take the average value of \( \mathbf{v}_f \) over all possible collision times \( t \) and all possible values of \( \mathbf{v}_i \). If we assume that the initial velocities are randomly distributed over all possible values, we see that the average value of \( \mathbf{v}_i \) is zero. The term \( \frac{qE}{m_e} t \) is the velocity change of the electron due to the electric field during one trip between atoms. The average value of the second term of Equation 27.13 is \( (qE/m_e)\tau \), where \( \tau \) is the average time interval between successive collisions. Because the average value of \( \mathbf{v}_f \) is equal to the drift velocity, we have

\[
\overline{\mathbf{v}}_f = \mathbf{v}_d = \frac{qE}{m_e} \tau
\]  

(27.14)

We can relate this expression for drift velocity to the current in the conductor. Substituting Equation 27.14 into Equation 27.6, we find that the magnitude of the current density is

\[
j = nq\mathbf{v}_d = \frac{nq^2E}{m_e} \tau
\]  

(27.15)

where \( n \) is the number of charge carriers per unit volume. Comparing this expression with Ohm’s law, \( j = \sigma E \), we obtain the following relationships for conductivity and resistivity of a conductor:

\[
\sigma = \frac{nq^2\tau}{m_e}
\]  

(27.16)

\[
\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}
\]  

(27.17)

According to this classical model, conductivity and resistivity do not depend on the strength of the electric field. This feature is characteristic of a conductor obeying Ohm’s law.

The average time interval \( \tau \) between collisions is related to the average distance between collisions \( \ell \) (that is, the mean free path; see Section 21.7) and the average speed \( \overline{v} \) through the expression

\[
\tau = \frac{\ell}{\overline{v}}
\]  

(27.18)

**Example 27.5  Electron Collisions in a Wire**

(A) Using the data and results from Example 27.1 and the classical model of electron conduction, estimate the average time interval between collisions for electrons in household copper wiring.

**Solution** From Equation 27.17, we see that

\[
\tau = \frac{m_e}{nq^2\rho}
\]

where \( \rho = 1.7 \times 10^{-8} \ \Omega \cdot m \) for copper and the carrier density is \( n = 8.49 \times 10^{28} \) electrons/m\(^3\) for the wire described in Example 27.1. Substitution of these values into the expression above gives

\[
\tau = \frac{9.11 \times 10^{-31} \text{ kg}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2 (1.7 \times 10^{-8} \text{ \Omega \cdot m })} = 2.5 \times 10^{-14} \text{ s}
\]

(B) Assuming that the average speed for free electrons in copper is \( 1.6 \times 10^6 \) m/s and using the result from part (A), calculate the mean free path for electrons in copper.
Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0 \left[ 1 + \alpha (T - T_0) \right]$$

(27.19)

where $\rho$ is the resistivity at some temperature $T$ (in degrees Celsius), $\rho_0$ is the resistivity at some reference temperature $T_0$ (usually taken to be 20°C), and $\alpha$ is the temperature coefficient of resistivity. From Equation 27.19, we see that the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0} \frac{\Delta \rho}{\Delta T}$$

(27.20)

where $\Delta \rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The temperature coefficients of resistivity for various materials are given in Table 27.1. Note that the unit for $\alpha$ is degrees Celsius$^{-1}$ [(°C)$^{-1}$]. Because resistance is proportional to resistivity (Eq. 27.11), we can write the variation of resistance as

$$R = R_0 \left[ 1 + \alpha (T - T_0) \right]$$

(27.21)

Use of this property enables us to make precise temperature measurements, as shown in Example 27.6.

Quick Quiz 27.6 When does a lightbulb carry more current: (a) just after it is turned on and the glow of the metal filament is increasing, or (b) after it has been on for a few milliseconds and the glow is steady?

Example 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

**Solution** Solving Equation 27.21 for $\Delta T$ and using the $\alpha$ value for platinum given in Table 27.1, we obtain

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8 \Omega - 50.0 \Omega}{[3.92 \times 10^{-3} \text{°C}^{-1}](50.0 \Omega)} = 137°C$$

Because $T_0 = 20.0°C$, we find that $T$, the temperature of the melting indium sample, is 157°C.

For metals like copper, resistivity is nearly proportional to temperature, as shown in Figure 27.10. However, a nonlinear region always exists at very low temperatures, and the resistivity usually reaches some finite value as the temperature approaches absolute zero. This residual resistivity near absolute zero is caused primarily by the...
collision of electrons with impurities and imperfections in the metal. In contrast, high-
temperature resistivity (the linear region) is predominantly characterized by collisions
between electrons and metal atoms.

Notice that three of the $\alpha$ values in Table 27.1 are negative; this indicates that the
resistivity of these materials decreases with increasing temperature (Fig. 27.11), which
is indicative of a class of materials called semiconductors. This behavior is due to an
increase in the density of charge carriers at higher temperatures.

Because the charge carriers in a semiconductor are often associated with impurity
atoms, the resistivity of these materials is very sensitive to the type and concentration of
such impurities. We shall return to the study of semiconductors in Chapter 43.

### 27.5 Superconductors

There is a class of metals and compounds whose resistance decreases to zero when they
are below a certain temperature $T_c$, known as the critical temperature. These
materials are known as superconductors. The resistance–temperature graph for a
superconductor follows that of a normal metal at temperatures above $T_c$ (Fig. 27.12).
When the temperature is at or below $T_c$, the resistivity drops suddenly to zero. This
phenomenon was discovered in 1911 by the Dutch physicist Heike Kamerlingh-Onnes
(1853–1926) as he worked with mercury, which is a superconductor below 4.2 K. Recent
measurements have shown that the resistivities of superconductors below their
$T_c$ values are less than $4 \times 10^{-25} \Omega \cdot m$—around $10^{17}$ times smaller than the resistivity
of copper and in practice considered to be zero.

Today thousands of superconductors are known, and as Table 27.3 illustrates, the
critical temperatures of recently discovered superconductors are substantially higher
than initially thought possible. Two kinds of superconductors are recognized. The
more recently identified ones are essentially ceramics with high critical temperatures,
whereas superconducting materials such as those observed by Kamerlingh-Onnes are
metals. If a room-temperature superconductor is ever identified, its impact on
technology could be tremendous.

The value of $T_c$ is sensitive to chemical composition, pressure, and molecular
structure. It is interesting to note that copper, silver, and gold, which are excellent
conductors, do not exhibit superconductivity.

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HgBa$_2$Ca$_2$Cu$_3$O$_8$</td>
<td>134</td>
</tr>
<tr>
<td>Tl–Ba–Ca–Cu–O</td>
<td>125</td>
</tr>
<tr>
<td>Bi–Sr–Ca–Cu–O</td>
<td>105</td>
</tr>
<tr>
<td>YBa$_2$Cu$_3$O$_7$</td>
<td>92</td>
</tr>
<tr>
<td>Nb$_3$Ge</td>
<td>23.2</td>
</tr>
<tr>
<td>Nb$_3$Sn</td>
<td>18.05</td>
</tr>
<tr>
<td>Nb</td>
<td>9.46</td>
</tr>
<tr>
<td>Pb</td>
<td>7.18</td>
</tr>
<tr>
<td>Hg</td>
<td>4.15</td>
</tr>
<tr>
<td>Sn</td>
<td>3.72</td>
</tr>
<tr>
<td>Al</td>
<td>1.19</td>
</tr>
<tr>
<td>Zn</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 27.3

Critical Temperatures for Various Superconductors
One of the truly remarkable features of superconductors is that once a current is set up in them, it persists without any applied potential difference (because $R = 0$). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important and useful application of superconductivity is in the development of superconducting magnets, in which the magnitudes of the magnetic field are about ten times greater than those produced by the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. Superconducting magnets are currently used in medical magnetic resonance imaging (MRI) units, which produce high-quality images of internal organs without the need for excessive exposure of patients to x-rays or other harmful radiation.

For further information on superconductivity, see Section 43.8.

### 27.6 Electrical Power

If a battery is used to establish an electric current in a conductor, there is a continuous transformation of chemical energy in the battery to kinetic energy of the electrons to internal energy in the conductor, resulting in an increase in the temperature of the conductor.

In typical electric circuits, energy is transferred from a source such as a battery, to some device, such as a lightbulb or a radio receiver. Let us determine an expression that will allow us to calculate the rate of this energy transfer. First, consider the simple circuit in Figure 27.13, where we imagine energy is being delivered to a resistor. (Resistors are designated by the circuit symbol $\square$.) Because the connecting wires also have resistance, some energy is delivered to the wires and some energy to the resistor. Unless noted otherwise, we shall assume that the resistance of the wires is so small compared to the resistance of the circuit element that we ignore the energy delivered to the wires.

Imagine following a positive quantity of charge $Q$ that is moving clockwise around the circuit in Figure 27.13 from point $a$ through the battery and resistor back to point $a$. We identify the entire circuit as our system. As the charge moves from $a$ to $b$ through the battery, the electric potential energy of the system increases by an amount $Q \Delta V$ while the chemical potential energy in the battery decreases by the same amount. (Recall from Eq. 25.9 that $\Delta U = q \Delta V$.) However, as the charge moves from $c$ to $d$ through the resistor, the system loses this electric potential energy during collisions of electrons with atoms in the resistor. In this process, the energy is transformed to internal energy corresponding to increased vibrational motion of the atoms in the resistor. Because we have neglected the resistance of the interconnecting wires, no energy transformation occurs for paths $bc$ and $da$. When the charge returns to point $a$, the net result is that some of the chemical energy in the battery has been delivered to the resistor and resides in the resistor as internal energy associated with molecular vibration.

The resistor is normally in contact with air, so its increased temperature will result in a transfer of energy by heat into the air. In addition, the resistor emits thermal

![Active Figure 27.13](image)

**Active Figure 27.13** A circuit consisting of a resistor of resistance $R$ and a battery having a potential difference $\Delta V$ across its terminals. Positive charge flows in the clockwise direction.

### Pitfall Prevention

#### 27.6 Misconceptions About Current

There are several common misconceptions associated with current in a circuit like that in Figure 27.13. One is that current comes out of one terminal of the battery and is then “used up” as it passes through the resistor, leaving current in only one part of the circuit. The truth is that the current is the same everywhere in the circuit. A related misconception has the current coming out of the resistor being smaller than that going in, because some of the current is “used up.” Another misconception has current coming out of both terminals of the battery, in opposite directions, and then “clashing” in the resistor, delivering the energy in this manner. This is not the case—the charges flow in the same rotational sense at all points in the circuit.
PITFALL PREVENTION

27.7 Charges Do Not Move All the Way Around a Circuit in a Short Time

Due to the very small magnitude of the drift velocity, it might take hours for a single electron to make one complete trip around the circuit. In terms of understanding the energy transfer in a circuit, however, it is useful to imagine a charge moving all the way around the circuit.

Power delivered to a device

Equation 27.23 described as the way around the circuit.

Energy Is Not Charges Do Not

It is commonly called heat sink connected to parts of the circuit to prevent these parts from reaching dangerously high temperatures. These are pieces of metal with many fins. The high thermal conductivity of the metal provides a rapid transfer of energy by heat away from the hot component, while the large number of fins provides a large surface area in contact with the air, so that energy can transfer by radiation and into the air by heat at a high rate.

Let us consider now the rate at which the system loses electric potential energy as the charge $Q$ passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt} (Q \Delta V) = \frac{dQ}{dt} \Delta V = I \Delta V$$

where $I$ is the current in the circuit. The system regains this potential energy when the charge passes through the battery, at the expense of chemical energy in the battery. The rate at which the system loses potential energy as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor. Thus, the power $P$, representing the rate at which energy is delivered to the resistor, is

$$P = I \Delta V$$

We derived this result by considering a battery delivering energy to a resistor. However, Equation 27.22 can be used to calculate the power delivered by a voltage source to any device carrying a current $I$ and having a potential difference $\Delta V$ between its terminals.

Using Equation 27.22 and the fact that $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms

$$P = I^2R = \frac{(\Delta V)^2}{R}$$

(27.23)

When $I$ is expressed in amperes, $\Delta V$ in volts, and $R$ in ohms, the SI unit of power is the watt, as it was in Chapter 7 in our discussion of mechanical power. The process by which power is lost as internal energy in a conductor of resistance $R$ is often called joule heating; this transformation is also often referred to as an $I^2R$ loss.

When transporting energy by electricity through power lines, such as those shown in the opening photograph for this chapter, we cannot make the simplifying assumption that the lines have zero resistance. Real power lines do indeed have resistance, and power is delivered to the resistance of these wires. Utility companies seek to minimize the power transformed to internal energy in the lines and maximize the energy delivered to the consumer. Because $P = I \Delta V$, the same amount of power can be transported either at high currents and low potential differences or at low currents and high potential differences. Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons. Copper wire is very expensive, and so it is cheaper to use high-resistance wire (that is, wire having a small cross-sectional area; see Eq. 27.11). Thus, in the expression for the power delivered to a resistor, $P = I^2R$, the resistance of the wire is fixed at a relatively high value for economic considerations. The $I^2R$ loss can be reduced by keeping the current $I$ as low as possible, which means transferring the energy at a high voltage. In some instances, power is transported at potential differences as great as 765 kV. Once the electricity reaches your city, the potential difference is usually reduced to 4 kV by a device called a transformer. Another

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4. This is another misuse of the word heat that is ingrained in our common language.

5. It is commonly called joule heating even though the process of heat does not occur. This is another example of incorrect usage of the word heat that has become entrenched in our language.
transformer drops the potential difference to 240 V before the electricity finally reaches your home. Of course, each time the potential difference decreases, the current increases by the same factor, and the power remains the same. We shall discuss transformers in greater detail in Chapter 33.

Demands on our dwindling energy supplies have made it necessary for us to be aware of the energy requirements of our electrical devices. Every electrical appliance carries a label that contains the information you need to calculate the appliance’s power requirements. In many cases, the power consumption in watts is stated directly, as it is on a lightbulb. In other cases, the amount of current used by the device and the potential difference at which it operates are given. This information and Equation 27.22 are sufficient for calculating the power requirement of any electrical device.

**Quick Quiz 27.7** The same potential difference is applied to the two lightbulbs shown in Figure 27.14. Which one of the following statements is true?
(a) The 30-W bulb carries the greater current and has the higher resistance.
(b) The 30-W bulb carries the greater current, but the 60-W bulb has the higher resistance.
(c) The 30-W bulb has the higher resistance, but the 60-W bulb carries the greater current.
(d) The 60-W bulb carries the greater current and has the higher resistance.

Figure 27.14 (Quick Quiz 27.7) These lightbulbs operate at their rated power only when they are connected to a 120-V source.

**Quick Quiz 27.8** For the two lightbulbs shown in Figure 27.15, rank the current values at points a through f, from greatest to least.

Figure 27.15 (Quick Quiz 27.8) Two lightbulbs connected across the same potential difference.

**Example 27.7 Power in an Electric Heater**

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

**Solution** Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120 \text{ V}}{8.00 \text{ Ω}} = 15.0 \text{ A}$$

We can find the power rating using the expression $P = I^2R$:

$$P = I^2R = (15.0 \text{ A})^2(8.00 \text{ Ω}) = 1.80 \times 10^3 \text{ W}$$

**What If?** What if the heater were accidentally connected to a 240-V supply? (This is difficult to do because the shape and orientation of the metal contacts in 240-V plugs are different from those in 120-V plugs.) How would this affect the current carried by the heater and the power rating of the heater?

**Answer** If we doubled the applied potential difference, Equation 27.8 tells us that the current would double. According to Equation 27.23, $P = (\Delta V)^2/R$, the power would be four times larger.
Example 27.8 Linking Electricity and Thermodynamics

(A) What is the required resistance of an immersion heater that will increase the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?

(B) Estimate the cost of heating the water.

Solution This example allows us to link our new understanding of power in electricity with our experience with specific heat in thermodynamics (Chapter 20). An immersion heater is a resistor that is inserted into a container of water. As energy is delivered to the immersion heater, raising its temperature, energy leaves the surface of the resistor by heat, going into the water. When the immersion heater reaches a constant temperature, the rate of energy delivered to the resistance by electrical transmission is equal to the rate of energy delivered by heat to the water.

(A) To simplify the analysis, we ignore the initial period during which the temperature of the resistor increases, and also ignore any variation of resistance with temperature. Thus, we imagine a constant rate of energy transfer for the entire 10.0 min. Setting the rate of energy delivered to the resistor equal to the rate of energy entering the water by heat, we have

\[ P = \frac{(\Delta V)^2}{R} = \frac{Q}{\Delta t} \]

where \( Q \) represents an amount of energy transfer by heat into the water and we have used Equation 27.23 to express the electrical power. The amount of energy transfer by heat necessary to raise the temperature of the water is given by Equation 20.4, \( Q = mc \Delta T \). Thus,

\[ R = \frac{(\Delta V)^2 \Delta t}{mc \Delta T} \]

Substituting the values given in the statement of the problem, we have

\[ R = \frac{(110 \text{ V})^2 (600 \text{ s})}{(1.50 \text{ kg})(4186 \text{ J/kg} \cdot \text{C})(50.0\degree \text{C} - 10.0\degree \text{C})} = \frac{28.9 \Omega}{110 \text{ V}} \]

(B) Because the energy transferred equals power multiplied by time interval, the amount of energy transferred is

\[ \Delta \text{ energy} = \frac{(\Delta V)^2}{R} \Delta t = \frac{(110 \text{ V})^2}{28.9 \Omega} (10.0 \text{ min}) \left( \frac{1 \text{ h}}{60.0 \text{ min}} \right) \]

\[ = 69.8 \text{ Wh} = 0.0698 \text{ kWh} \]

If the energy is purchased at an estimated price of 10.0¢ per kilowatt-hour, the cost is

\[ \text{Cost} = (0.0698 \text{ kWh}) (0.100/\text{kWh}) = 0.00698 \]

\[ \approx 0.7 \text{¢} \]

At the Interactive Worked Example link at http://www.pse6.com, you can explore the heating of the water.

Example 27.9 Current in an Electron Beam

In a certain particle accelerator, electrons emerge with an energy of 40.0 MeV (1 MeV = 1.60 × 10^{-13} J). The electrons emerge not in a steady stream but rather in pulses at the rate of 250 pulses/s. This corresponds to a time interval between pulses of 4.00 ms (Fig. 27.16). Each pulse has a duration of 200 ns, and the electrons in the pulse constitute a current of 250 mA. The current is zero between pulses.

(A) How many electrons are delivered by the accelerator per pulse?

Solution We use Equation 27.2 in the form \( dQ = I \, dt \) and integrate to find the charge per pulse. While the pulse is on, the current is constant; thus,

\[ Q_{\text{pulse}} = \int dt = I \Delta t = (250 \times 10^{-3} \text{ A})(200 \times 10^{-9} \text{ s}) \]

\[ = 5.00 \times 10^{-8} \text{ C} \]

Dividing this quantity of charge per pulse by the electronic charge gives the number of electrons per pulse:
(B) What is the average current per pulse delivered by the accelerator?

**Solution** Average current is given by Equation 27.1, \( I_{av} = \frac{Q_{pulse}}{\Delta t} \). Because the time interval between pulses is 4.00 ms, and because we know the charge per pulse from part (A), we obtain

\[
I_{av} = \frac{5.00 \times 10^{-8} \text{ C}}{4.00 \times 10^{-3} \text{ s}} = 12.5 \mu\text{A}
\]

This represents only 0.005% of the peak current, which is 250 mA.

(C) What is the peak power delivered by the electron beam?

**Solution** By definition, power is energy delivered per unit time interval. Thus, the peak power is equal to the energy delivered by a pulse divided by the pulse duration:

\[
\mathcal{P}_{\text{peak}} = \frac{\text{pulse energy}}{\text{pulse duration}} = \frac{(3.13 \times 10^{11} \text{ electrons/pulse})(40.0 \text{ MeV/electron})}{2.00 \times 10^{-7} \text{ s/pulse}} \times \left( \frac{1.60 \times 10^{-13} \text{ J}}{1 \text{ MeV}} \right)
\]

\[
= 1.00 \times 10^7 \text{ W} = 10.0 \text{ MW}
\]

We could also compute this power directly. We assume that each electron has zero energy before being accelerated. Thus, by definition, each electron must go through a potential difference of 40.0 MV to acquire a final energy of 40.0 MeV. Hence, we have

\[
(2) \quad \mathcal{P}_{\text{peak}} = I_{\text{peak}} \Delta V
\]

\[
= (250 \times 10^{-3} \text{ A})(40.0 \times 10^6 \text{ V})
\]

\[
= 10.0 \text{ MW}
\]

**What If?** What if the requested quantity in part (C) were the average power rather than the peak power?

**Answer** Instead of Equation (1), we would use the time interval between pulses rather than the duration of a pulse:

\[
\mathcal{P}_{\text{av}} = \frac{\text{pulse energy}}{\text{time interval between pulses}} = \frac{(3.13 \times 10^{11} \text{ electrons/pulse})(40.0 \text{ MeV/electron})}{4.00 \times 10^{-3} \text{ s/pulse}}
\]

\[
\times \left( \frac{1.60 \times 10^{-13} \text{ J}}{1 \text{ MeV}} \right)
\]

\[
= 500 \text{ W}
\]

Instead of Equation (2), we would use the average current found in part (B):

\[
\mathcal{P}_{\text{av}} = I_{av} \Delta V = (12.5 \times 10^{-6} \text{ A})(40.0 \times 10^6 \text{ V})
\]

\[
= 500 \text{ W}
\]

Notice that these two calculations agree with each other and that the average power is much lower than the peak power.

---

**SUMMARY**

The **electric current** \( I \) in a conductor is defined as

\[
I = \frac{dQ}{dt}
\]

(27.2)

where \( dQ \) is the charge that passes through a cross section of the conductor in a time interval \( dt \). The SI unit of current is the **ampere** (A), where 1 A = 1 C/s.

The average current in a conductor is related to the motion of the charge carriers through the relationship

\[
I_{av} = nqv_d A
\]

(27.4)

where \( n \) is the density of charge carriers, \( q \) is the charge on each carrier, \( v_d \) is the drift speed, and \( A \) is the cross-sectional area of the conductor.

The magnitude of the **current density** \( J \) in a conductor is the current per unit area:

\[
J = \frac{I}{A} = nqv_d
\]

(27.5)
The current density in an ohmic conductor is proportional to the electric field according to the expression

\[ J = \sigma E \]  
(27.7)

The proportionality constant \( \sigma \) is called the **conductivity** of the material of which the conductor is made. The inverse of \( \sigma \) is known as **resistivity** \( \rho \) (that is, \( \rho = 1/\sigma \)). Equation 27.7 is known as **Ohm’s law**, and a material is said to obey this law if the ratio of its current density \( J \) to its applied electric field \( E \) is a constant that is independent of the applied field.

The **resistance** \( R \) of a conductor is defined as

\[ R = \frac{\Delta V}{I} \]  
(27.8)

where \( \Delta V \) is the potential difference across it, and \( I \) is the current it carries.

The SI unit of resistance is volts per ampere, which is defined to be 1 **ohm** (Ω); that is, 1 Ω = 1 V/A. If the resistance is independent of the applied potential difference, the conductor obeys Ohm’s law.

For a uniform block of material of cross sectional area \( A \) and length \( \ell \), the resistance over the length \( \ell \) is

\[ R = \rho \frac{\ell}{A} \]  
(27.11)

where \( \rho \) is the resistivity of the material.

In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a **drift velocity** \( v_d \) that is opposite the electric field and given by the expression

\[ v_d = \frac{qE}{m_e \tau} \]  
(27.14)

where \( \tau \) is the average time interval between electron–atom collisions, \( m_e \) is the mass of the electron, and \( q \) is its charge. According to this model, the resistivity of the metal is

\[ \rho = \frac{m_e}{n q^2 \tau} \]  
(27.17)

where \( n \) is the number of free electrons per unit volume.

The resistivity of a conductor varies approximately linearly with temperature according to the expression

\[ \rho = \rho_0 [1 + \alpha(T - T_0)] \]  
(27.19)

where \( \alpha \) is the **temperature coefficient of resistivity** and \( \rho_0 \) is the resistivity at some reference temperature \( T_0 \).

If a potential difference \( \Delta V \) is maintained across a circuit element, the **power**, or rate at which energy is supplied to the element, is

\[ \mathcal{P} = I \Delta V \]  
(27.22)

Because the potential difference across a resistor is given by \( \Delta V = IR \), we can express the power delivered to a resistor in the form

\[ \mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \]  
(27.23)

The energy delivered to a resistor by electrical transmission appears in the form of internal energy in the resistor.
1. In an analogy between electric current and automobile traffic flow, what would correspond to charge? What would correspond to current?

2. Newspaper articles often contain a statement such as “10,000 volts of electricity surged through the victim’s body.” What is wrong with this statement?

3. What factors affect the resistance of a conductor?

4. What is the difference between resistance and resistivity?

5. Two wires A and B of circular cross section are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross-sectional areas? How do their radii compare?

6. Do all conductors obey Ohm’s law? Give examples to justify your answer.

7. We have seen that an electric field must exist inside a conductor that carries a current. How is it possible in view of the fact that in electrostatics we concluded that the electric field must be zero inside a conductor?

8. A very large potential difference is not necessarily required to produce long sparks in air. With a device called Jacob's ladder, a potential difference of about 10 kV produces an electric arc a few millimeters long between the bottom ends of two curved rods that project upward from the power supply. (The device is seen in classic mad-scientist horror movies and in Figure Q27.8.) The arc rises, climbing the rods and getting longer and longer. It disappears when it reaches the top; then a new spark immediately forms at the bottom and the process repeats. Explain these phenomena. Why does the arc rise? Why does a new arc appear only after the previous one is gone?

9. When the voltage across a certain conductor is doubled, the current is observed to increase by a factor of three. What can you conclude about the conductor?

10. In the water analogy of an electric circuit, what corresponds to the power supply, resistor, charge, and potential difference?

11. Use the atomic theory of matter to explain why the resistance of a material should increase as its temperature increases.

12. Why might a “good” electrical conductor also be a “good” thermal conductor?

13. How does the resistance for copper and for silicon change with temperature? Why are the behaviors of these two materials different?

14. Explain how a current can persist in a superconductor without any applied voltage.

15. What single experimental requirement makes superconducting devices expensive to operate? In principle, can this limitation be overcome?

16. What would happen to the drift velocity of the electrons in a wire and to the current in the wire if the electrons could move freely without resistance through the wire?

17. If charges flow very slowly through a metal, why does it not require several hours for a light to come on when you throw a switch?

18. In a conductor, changes in the electric field that drives the electrons through the conductor propagate with a speed close to the speed of light, although the drift velocity of the electrons is very small. Explain how these statements can both be true. Does one particular electron move from one end of the conductor to the other?

19. Two conductors of the same length and radius are connected across the same potential difference. One conductor has twice the resistance of the other. To which conductor is more power delivered?

20. Two light bulbs both operate from 120 V. One has a power of 25 W and the other 100 W. Which bulb has higher resistance? Which bulb carries more current?

21. Car batteries are often rated in ampere-hours. Does this designate the amount of current, power, energy, or charge that can be drawn from the battery?

22. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1000 W?
Section 27.1 Electric Current

1. In a particular cathode ray tube, the measured beam current is 50.0 μA. How many electrons strike the tube screen every 40.0 s?

2. A teapot with a surface area of 700 cm² is to be silver plated. It is attached to the negative electrode of an electrolytic cell containing silver nitrate (Ag⁺ NO₃⁻). If the cell is powered by a 12.0-V battery and has a resistance of 1.80 Ω, how long does it take for a 0.133-mm layer of silver to build up on the teapot? (The density of silver is 10.5 × 10³ kg/m³.)

3. Suppose that the current through a conductor decreases exponentially with time according to the equation \( I(t) = I₀e^{-t/τ} \) where \( I₀ \) is the initial current (at \( t = 0 \)), and \( τ \) is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between \( t = 0 \) and \( t = τ? \) (b) How much charge passes this point between \( t = 0 \) and \( t = τ? \) (c) **What If?** How much charge passes this point between \( t = 0 \) and \( t = ∞? \)

4. In the Bohr model of the hydrogen atom, an electron in the lowest energy state follows a circular path 5.29 × 10⁻¹¹ m from the proton. (a) Show that the speed of the electron is 2.19 × 10⁶ m/s. (b) What is the effective current associated with this orbiting electron?

5. A small sphere that carries a charge \( q \) is whirled in a circle at the end of an insulating string. The angular frequency of rotation is \( ω \). What average current does this rotating charge represent?

6. The quantity of charge \( q \) (in coulombs) that has passed through a surface of area 2.00 cm² varies with time according to the equation \( q = 4t^3 + 5t + 6 \), where \( t \) is in seconds. (a) What is the instantaneous current through the surface at \( t = 1.00 \) s? (b) What is the value of the current density?

7. An electric current is given by the expression \( I(t) = 100 \sin(120πt) \), where \( I \) is in amperes and \( t \) is in seconds. What is the total charge carried by the current from \( t = 0 \) to \( t = (1/240) \) s?

8. Figure P27.8 represents a section of a circular conductor of nonuniform diameter carrying a current of 5.00 A. The radius of cross section \( A_1 \) is 0.400 cm. (a) What is the magnitude of the current density across \( A_1? \) (b) If the current density across \( A_2 \) is one-fourth the value across \( A_1 \), what is the radius of the conductor at \( A_2? \)

The electron beam emerging from a certain high-energy electron accelerator has a circular cross section of radius 1.00 mm. (a) The beam current is 8.00 μA. Find the current density in the beam, assuming that it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as \( c = 3.00 \times 10^8 \) m/s with negligible error. Find the electron density in the beam. (c) How long does it take for Avogadro’s number of electrons to emerge from the accelerator?

10. A Van de Graaff generator produces a beam of 2.00-MeV deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is 10.0 μA, how far apart are the deuterons? (b) Is the electric force of repulsion among them a significant factor in beam stability? Explain.

11. An aluminum wire having a cross-sectional area of 4.00 × 10⁻⁷ m² carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm³. Assume that one conduction electron is supplied by each atom.

Section 27.2 Resistance

12. Calculate the current density in a gold wire at 20°C, if an electric field of 0.740 V/m exists in the wire.

13. A lighthouse has a resistance of 240 Ω when operating with a potential difference of 120 V across it. What is the current in the lighthouse?

14. A resistor is constructed of a carbon rod that has a uniform cross-sectional area of 5.00 mm². When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of 4.00 × 10⁻³ A. Find (a) the resistance of the rod and (b) the rod’s length.

15. A 0.900-V potential difference is maintained across a 1.50-m-length of tungsten wire that has a cross-sectional area of 0.600 mm². What is the current in the wire?

16. A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m. What is the resistivity of the material?

17. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of \( R = 0.500 \) Ω, and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?

18. Gold is the most ductile of all metals. For example, one gram of gold can be drawn into a wire 2.40 km long. What is the resistance of such a wire at 20°C? You can find the necessary reference information in this textbook.
19. (a) Make an order-of-magnitude estimate of the resistance between the ends of a rubber band. (b) Make an order-of-magnitude estimate of the resistance between the ‘heads’ and ‘tails’ sides of a penny. In each case state what quantities you take as data and the values you measure or estimate for them. (c) WARNING! Do not try this at home! What is the order of magnitude of the current that each would carry if it were connected across a 120-V power supply?

20. A solid cube of silver (density = 10.5 g/cm³) has a mass of 90.0 g. (a) What is the resistance between opposite faces of the cube? (b) Assume each silver atom contributes one conduction electron. Find the average drift speed of electrons when a potential difference of 1.00 × 10⁻³ V is applied to opposite faces. The atomic number of silver is 47, and its molar mass is 107.87 g/mol.

21. A metal wire of resistance R is cut into three equal pieces that are then connected side by side to form a new wire the length of which is equal to one-third the original length. What is the resistance of this new wire?

22. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

23. A current density of 6.00 × 10⁻¹³ A/m² exists in the atmosphere at a location where the electric field is 100 V/m. Calculate the electrical conductivity of the Earth’s atmosphere in this region.

24. The rod in Figure P27.24 is made of two materials. The figure is not drawn to scale. Each conductor has a square cross section 3.00 mm on a side. The first material has a resistivity of 4.00 × 10⁻³ Ω·m and is 25.0 cm long, while the second material has a resistivity of 6.00 × 10⁻³ Ω·m and is 40.0 cm long. What is the resistance between the ends of the rod?

![Figure P27.24](image)

Section 27.3 A Model for Electrical Conduction

25. If the magnitude of the drift velocity of free electrons in a copper wire is 7.84 × 10⁻⁴ m/s, what is the electric field in the conductor?

26. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density? (b) current density? (c) electron drift velocity? (d) average time interval between collisions?

27. Use data from Example 27.1 to calculate the collision mean free path of electrons in copper. Assume the average thermal speed of conduction electrons is 8.60 × 10⁵ m/s.

Section 27.4 Resistance and Temperature

28. While taking photographs in Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.000 A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is −88.0°C? Assume that no change occurs in the wire’s shape and size.

29. A certain lightbulb has a tungsten filament with a resistance of 19.0 Ω when cold and 140 Ω when hot. Assume that the resistivity of tungsten varies linearly with temperature even over the large temperature range involved here, and find the temperature of the hot filament. Assume the initial temperature is 20.0°C.

30. A carbon wire and a Nichrome wire are connected in series, so that the same current exists in both wires. If the combination has a resistance of 10.0 Ω at 0°C, what is the resistance of each wire at 0°C so that the resistance of the combination does not change with temperature? The total or equivalent resistance of resistors in series is the sum of their individual resistances.

31. An aluminum wire with a diameter of 0.100 mm has a uniform electric field of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C. Assume one free electron per atom. (a) Use the information in Table 27.1 and determine the resistivity. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2.00-m length of the wire to produce the stated electric field?

32. Review problem. An aluminum rod has a resistance of 1.234 Ω at 20.0°C. Calculate the resistance of the rod at 120°C by accounting for the changes in both the resistivity and the dimensions of the rod.

33. What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0°C to 50.0°C?

34. The resistance of a platinum wire is to be calibrated for low-temperature measurements. A platinum wire with resistance 1.00 Ω at 20.0°C is immersed in liquid nitrogen at 77 K (≈ 196°C). If the temperature response of the platinum wire is linear, what is the expected resistance of the platinum wire at −196°C? (αplatinum = 3.92 × 10⁻³/°C)

35. The temperature of a sample of tungsten is raised while a sample of copper is maintained at 20.0°C. At what temperature will the resistivity of the tungsten be four times that of the copper?

Section 27.6 Electrical Power

36. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

37. A Van de Graaff generator (see Figure 25.29) is operating so that the potential difference between the high-voltage electrode B and the charging needles at A is 15.0 kV. Calculate the power required to drive the belt against electrical forces at an instant when the effective current delivered to the high-voltage electrode is 500 μA.

38. In a hydroelectric installation, a turbine delivers 1 500 hp to a generator, which in turn transfers 80.0% of the mechanical energy out by electrical transmission. Under
these conditions, what current does the generator deliver at a terminal potential difference of 2,000 V?

59. What is the required resistance of an immersion heater that increases the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?

40. One rechargeable battery of mass 13.0 g delivers to a CD player an average current of 18.0 mA at 1.60 V for 2.40 h before the battery needs to be recharged. The recharger maintains a potential difference of 2.50 V across the battery and delivers a charging current of 13.5 mA for 4.20 h. (a) What is the efficiency of the battery as an energy storage device? (b) How much internal energy is produced in the battery during one charge-discharge cycle? (c) If the battery is surrounded by ideal thermal insulation and has an overall effective specific heat of 975 J/kg°C, by how much will its temperature increase during the cycle?

41. Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W lightbulb increase? Assume that its resistance does not change.

42. A 500-W heating coil designed to operate from 110 V is made of Nichrome wire 0.500 mm in diameter. (a) Assuming that the resistivity of the Nichrome remains constant at its 20.0°C value, find the length of wire used. (b) What If? Now consider the variation of resistivity with temperature. What power will the coil of part (a) actually deliver when it is heated to 1200°C?

43. A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the magnitude of the electric field in the wire, and (b) the power delivered to it? (c) What If? If the temperature is increased to 340°C and the voltage across the wire remains constant, what is the power delivered?

44. Batteries are rated in terms of ampere-hours (A · h). For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A · h. (a) What is the total energy, in kilowatt-hours, stored in a 12.0-V battery rated at 55.0 A · h? (b) At $0.060/0 per kilowatt-hour, what is the value of the electricity produced by this battery?

45. A 10.0-V battery is connected to a 120-Ω resistor. Ignoring the internal resistance of the battery, calculate the power delivered to the resistor.

46. Residential building codes typically require the use of 12-gauge copper wire (diameter 0.205 3 cm) for wiring receptacles. Such circuits carry currents as large as 20 A. A wire of smaller diameter (with a higher gauge number) could carry this much current, but the wire could rise to a high temperature and cause a fire. (a) Calculate the rate at which internal energy is produced in 1.00 m of 12-gauge copper wire carrying a current of 20.0 A. (b) What If? Repeat the calculation for an aluminum wire. Would a 12-gauge aluminum wire be as safe as a copper wire?

47. An 11.0-W energy-efficient fluorescent lamp is designed to produce the same illumination as a conventional 40.0-W incandescent lightbulb. How much money does the user of the energy-efficient lamp save during 100 hours of use? Assume a cost of $0.080/0/kWh for energy from the power company.

48. We estimate that 270 million plug-in electric clocks are in the United States, approximately one clock for each person. The clocks convert energy at the average rate 2.50 W. To supply this energy, how many metric tons of coal are burned per hour in coal-fired electric generating plants that are, on average, 25.0% efficient? The heat of combustion for coal is 33.0 MJ/kg.

49. Compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line. Assume the cost of energy from the power company is $0.060/0/kWh.

50. Review problem. The heating element of a coffee maker operates at 120 V and carries a current of 2.00 A. Assuming that the water absorbs all of the energy delivered to the resistor, calculate how long it takes to raise the temperature of 0.500 kg of water from room temperature (23.0°C) to the boiling point.

51. A certain toaster has a heating element made of Nichrome wire. When the toaster is first connected to a 120-V source (and the wire is at a temperature of 20.0°C), the initial current is 1.80 A. However, the current begins to decrease as the heating element warms up. When the toaster reaches its final operating temperature, the current drops to 1.53 A. (a) Find the power delivered to the toaster when it is at its operating temperature. (b) What is the final temperature of the heating element?

52. The cost of electricity varies widely throughout the United States; $0.120/kWh is one typical value. At this unit price, calculate the cost of (a) leaving a 40.0-W porch light on for two weeks while you are on vacation, (b) making a piece of dark toast in 3.00 min with a 970-W toaster, and (c) drying a load of clothes in 40.0 min in a 5 200-W dryer.

53. Make an order-of-magnitude estimate of the cost of one person’s routine use of a hair dryer for 1 yr. If you do not use a blow dryer yourself, observe or interview someone who does. State the quantities you estimate and their values.

Additional Problems

54. One lightbulb is marked ‘25 W 120 V,’ and another ‘100 W 120 V;’ this means that each bulb has its respective power delivered to it when plugged into a constant 120-V potential difference. (a) Find the resistance of each bulb. (b) How long does it take for 1.00 C to pass through the dim bulb? Is the charge different in any way upon its exit from the bulb versus its entry? (c) How long does it take for 1.00 J to pass through the dim bulb? By what mechanisms does this energy enter and exit the bulb? (d) Find how much it costs to run the dim bulb continuously for 30.0 days if the electric company sells its product at $0.070/0 per kWh. What product does the electric company sell? What is its price for one SI unit of this quantity?

55. A charge Q is placed on a capacitor of capacitance C. The capacitor is connected into the circuit shown in Figure P27.55, with an open switch, a resistor, and an initially uncharged capacitor of capacitance 3C. The switch is then closed and the circuit comes to equilibrium. In terms of Q and C, find (a) the final potential difference between the plates of each capacitor, (b) the charge on each capacitor,
56. A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1 000 A. If the conductor is copper wire with a free charge density of \(8.49 \times 10^{28}\) electrons/m\(^3\), how long does it take one electron to travel the full length of the line?

57. A more general definition of the temperature coefficient of resistivity is

\[ \alpha = \frac{1}{\rho} \frac{d\rho}{dT} \]

where \(\rho\) is the resistivity at temperature \(T\). (a) Assuming that \(\alpha\) is constant, show that

\[ \rho = \rho_0 e^{\alpha(T - T_0)} \]

where \(\rho_0\) is the resistivity at temperature \(T_0\). (b) Using the series expansion \(e^x = 1 + x + x^2/2! + x^3/3! + \ldots\) for \(x \ll 1\), show that the resistivity is given approximately by the expression

\[ \rho = \rho_0[1 + \alpha(T - T_0)] \]

for \(\alpha(T - T_0) \ll 1\).

58. A high-voltage transmission line carries 1 000 A starting at 700 kV for a distance of 100 mi. If the resistance in the wire is 0.500 \(\Omega/mi\), what is the power loss due to resistive losses?

59. An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses 30-gauge wire, which has a cross-sectional area of \(7.50 \times 10^{-8}\) m\(^2\). The student measures the potential difference across the wire and the current in the wire with a voltmeter and an ammeter, respectively. For each of the measurements given in the table taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. What is the average value of the resistivity, and how does this value compare with the value given in Table 27.1?

<table>
<thead>
<tr>
<th>(L) (m)</th>
<th>(\Delta V) (V)</th>
<th>(I) (A)</th>
<th>(R) ((\Omega))</th>
<th>(\rho) ((\Omega) · m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.540</td>
<td>5.22</td>
<td>0.500</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>1.028</td>
<td>5.82</td>
<td>0.276</td>
<td>0.187</td>
<td></td>
</tr>
<tr>
<td>1.543</td>
<td>5.94</td>
<td>0.187</td>
<td>0.187</td>
<td></td>
</tr>
</tbody>
</table>

60. An electric utility company supplies a customer’s house from the main power lines (120 V) with two copper wires, each of which is 50.0 m long and has a resistance of 0.108 \(\Omega\) per 300 m. (a) Find the voltage at the customer’s house for a load current of 110 A. For this load current, find (b) the power the customer is receiving and (c) the electric power lost in the copper wires.

61. A straight cylindrical wire lying along the \(x\) axis has a length of 0.500 m and a diameter of 0.200 mm. It is made of a material that obeys Ohm’s law with a resistivity of \(\rho = 4.00 \times 10^{-8}\) \(\Omega\) · m. Assume that a potential of 4.00 V is maintained at \(x = 0\), and that \(V = 0\) at \(x = 0.500\) m. Find (a) the electric field \(E\) in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density \(J\) in the wire. Express vectors in vector notation. (c) Show that \(E = J\rho\).

62. A straight cylindrical wire lying along the \(x\) axis has a length \(L\) and a diameter \(d\). It is made of a material that obeys Ohm’s law with a resistivity \(\rho\). Assume that potential \(V\) is maintained at \(x = 0\), and that the potential is zero at \(x = L\). In terms of \(L, d, V, \rho,\) and physical constants, derive expressions for (a) the electric field in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density in the wire. Express vectors in vector notation. (e) Prove that \(E = J\rho\).

63. The potential difference across the filament of a lamp is maintained at a constant level while equilibrium temperature is being reached. It is observed that the steady-state current in the lamp is only one tenth of the current drawn by the lamp when it is first turned on. If the temperature coefficient of resistivity for the lamp at 20.0°C is 0.004 50 (°C)\(^{-1}\), and if the resistivity increases linearly with increasing temperature, what is the final operating temperature of the filament?

64. The current in a resistor decreases by 3.00 A when the voltage applied across the resistor decreases from 12.0 V to 6.00 V. Find the resistance of the resistor.

65. An electric car is designed to run off a bank of 12.0-V batteries with total energy storage of 2.00 \(\times 10^7\) J. (a) If the electric motor draws 8.00 kW, what is the current delivered to the motor? (b) If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, how far will the car travel before it is out of juice?

66. Review problem. When a straight wire is heated, its resistance is given by \(R = R_0[1 + \alpha(T - T_0)]\) according to Equation 27.21, where \(\alpha\) is the temperature coefficient of resistivity. (a) Show that a more precise result, one that includes the fact that the length and area of the wire change when heated, is

\[ R = \frac{R_0[1 + \alpha(T - T_0)][1 + \alpha'(T - T_0)]}{[1 + 2\alpha'(T - T_0)]} \]

where \(\alpha'\) is the coefficient of linear expansion (see Chapter 19). (b) Compare these two results for a 2.00-m-long copper wire of radius 0.100 mm, first at 20.0°C and then heated to 100.0°C.

67. The temperature coefficients of resistivity in Table 27.1 were determined at a temperature of 20°C. What would they be at 0°C? Note that the temperature coefficient of resistivity at 20°C satisfies \(\rho = \rho_0[1 + \alpha(T - T_0)]\), where \(\rho_0\) is the resistivity of the material at \(T_0 = 20°C\). The temperature
coefficient of resistivity \( \alpha' \) at 0°C must satisfy the expression
\[ \rho = \rho_0' [1 + \alpha'T], \]
where \( \rho_0' \) is the resistivity of the material at 0°C.

68. An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water a pair of concentric metallic cylinders (Fig. P27.68) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius \( r_a \), outer radius \( r_b \), and length \( L \) much larger than \( r_b \). The scientist applies a potential difference \( \Delta V \) between the inner and outer surfaces, producing an outward radial current \( I \). Let \( \rho \) represent the resistivity of the water. (a) Find the resistance of the water between the cylinders in terms of \( L \), \( \rho \), \( r_a \), and \( r_b \). (b) Express the resistivity of the water in terms of the measured quantities \( L \), \( r_a \), \( r_b \), \( \Delta V \), and \( I \).

69. In a certain stereo system, each speaker has a resistance of 4.00 \( \Omega \). The system is rated at 60.0 W in each channel, and each speaker circuit includes a fuse rated 4.00 A. Is this system adequately protected against overload? Explain your reasoning.

70. A close analogy exists between the flow of energy by heat because of a temperature difference (see Section 20.7) and the flow of electric charge because of a potential difference. The energy \( dQ \) and the electric charge \( dq \) can both be transported by free electrons in the conducting material. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness \( dx \), area \( A \), and electrical conductivity \( \sigma \), with a potential difference \( dV \) between opposite faces. Show that the current \( I = dq/dt \) is given by the equation on the left below:

\[
\frac{dq}{dt} = \sigma A \frac{dV}{dx}
\]

\[
\frac{dQ}{dt} = kA \frac{dT}{dx}
\]

In the analogous thermal conduction equation on the right, the rate of energy flow \( dQ/dt \) (in SI units of joules per second) is due to a temperature gradient \( dT/dx \), in a material of thermal conductivity \( k \). State analogous rules relating the direction of the electric current to the change in potential, and relating the direction of energy flow to the change in temperature.

71. Material with uniform resistivity \( \rho \) is formed into a wedge as shown in Figure P27.71. Show that the resistance between face A and face B of this wedge is

\[
R = \rho \frac{L}{w(y_2 - y_1)} \ln \left( \frac{y_2}{y_1} \right)
\]

72. A material of resistivity \( \rho \) is formed into the shape of a truncated cone of altitude \( h \) as shown in Figure P27.72. The bottom end has radius \( b \), and the top end has radius \( a \). Assume that the current is distributed uniformly over any circular cross section of the cone, so that the current density does not depend on radial position. (The current density does vary with position along the axis of the cone.) Show that the resistance between the two ends is described by the expression

\[
R = \frac{\rho}{\pi} \left( \frac{h}{ab} \right)
\]

73. The dielectric material between the plates of a parallel-plate capacitor always has some nonzero conductivity \( \sigma \). Let \( A \) represent the area of each plate and \( d \) the distance between them. Let \( \kappa \) represent the dielectric constant of the material. (a) Show that the resistance \( R \) and the capacitance \( C \) of the capacitor are related by

\[
RC = \frac{\kappa \varepsilon_0}{\sigma}
\]

(b) Find the resistance between the plates of a 14.0-nF capacitor with a fused quartz dielectric.

74. The current–voltage characteristic curve for a semiconductor diode as a function of temperature \( T \) is given by the equation

\[
I = I_0 (e^{\Delta V/k_BT} - 1)
\]
Here the first symbol $e$ represents Euler’s number, the base of natural logarithms. The second $e$ is the charge on the electron. The $k_B$ stands for Boltzmann’s constant, and $T$ is the absolute temperature. Set up a spreadsheet to calculate $I$ and $R = \Delta V/I$ for $\Delta V = 0.400\, \text{V}$ to $0.600\, \text{V}$ in increments of $0.005\, \text{V}$. Assume $I_0 = 1.00\, \text{nA}$. Plot $R$ versus $\Delta V$ for $T = 280\, \text{K}$, $300\, \text{K}$, and $320\, \text{K}$.

75. Review problem. A parallel-plate capacitor consists of square plates of edge length $\ell$ that are separated by a distance $d$, where $d \ll \ell$. A potential difference $\Delta V$ is maintained between the plates. A material of dielectric constant $\kappa$ fills half of the space between the plates. The dielectric slab is now withdrawn from the capacitor, as shown in Figure P27.75. (a) Find the capacitance when the left edge of the dielectric is at a distance $x$ from the center of the capacitor. (b) If the dielectric is removed at a constant speed $v$, what is the current in the circuit as the dielectric is being withdrawn?

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**Figure P27.75**

**Answers to Quick Quizzes**

27.1 d, b = c, a. The current in part (d) is equivalent to two positive charges moving to the left. Parts (b) and (c) each represent four positive charges moving in the same direction because negative charges moving to the left are equivalent to positive charges moving to the right. The current in part (a) is equivalent to five positive charges moving to the right.

27.2 (b). The currents in the two paths add numerically to equal the current coming into the junction, without regard for the directions of the two wires coming out of the junction. This is indicative of scalar addition. Even though we can assign a direction to a current, it is not a vector. This suggests a deeper meaning for vectors besides that of a quantity with magnitude and direction.

27.3 (a). The current in each section of the wire is the same even though the wire constricts. As the cross-sectional area $A$ decreases, the drift velocity must increase in order for the constant current to be maintained, in accordance with Equation 27.4. As $A$ decreases, Equation 27.11 tells us that $R$ increases.

27.4 (b). The doubling of the radius causes the area $A$ to be four times as large, so Equation 27.11 tells us that the resistance decreases.

27.5 (b). The slope of the tangent to the graph line at a point is the reciprocal of the resistance at that point. Because the slope is increasing, the resistance is decreasing.

27.6 (a). When the filament is at room temperature, its resistance is low, and hence the current is relatively large. As the filament warms up, its resistance increases, and the current decreases. Older lightbulbs often fail just as they are turned on because this large initial current “spike” produces rapid temperature increase and mechanical stress on the filament, causing it to break.

27.7 (c). Because the potential difference $\Delta V$ is the same across the two bulbs and because the power delivered to a conductor is $\mathcal{P} = I\Delta V$, the 60-W bulb, with its higher power rating, must carry the greater current. The 30-W bulb has the higher resistance because it draws less current at the same potential difference.

27.8 $I_a = I_b > I_c = I_d > I_e = I_f$. The current $I_a$ leaves the positive terminal of the battery and then splits to flow through the two bulbs; thus, $I_a = I_c + I_e$. From Quick Quiz 27.7, we know that the current in the 60-W bulb is greater than that in the 30-W bulb. Because charge does not build up in the bulbs, we know that the same amount of charge flowing into a bulb from the left must flow out on the right; consequently, $I_e = I_d$ and $I_c = I_f$. The two currents leaving the bulbs recombine to form the current back into the battery, $I_f + I_d = I_b$. 

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Answers to Quick Quizzes