All of these devices are capacitors, which store electric charge and energy. A capacitor is one type of circuit element that we can combine with others to make electric circuits. (Paul Silverman/Fundamental Photographs)
In this chapter, we will introduce the first of three simple circuit elements that can be connected with wires to form an electric circuit. Electric circuits are the basis for the vast majority of the devices that we use in current society. We shall discuss capacitors—devices that store electric charge. This discussion will be followed by the study of resistors in Chapter 27 and inductors in Chapter 32. In later chapters, we will study more sophisticated circuit elements such as diodes and transistors.

Capacitors are commonly used in a variety of electric circuits. For instance, they are used to tune the frequency of radio receivers, as filters in power supplies, to eliminate sparking in automobile ignition systems, and as energy-storing devices in electronic flash units.

A capacitor consists of two conductors separated by an insulator. The capacitance of a given capacitor depends on its geometry and on the material—called a dielectric—that separates the conductors.

26.1 Definition of Capacitance

Consider two conductors carrying charges of equal magnitude and opposite sign, as shown in Figure 26.1. Such a combination of two conductors is called a capacitor. The conductors are called plates. A potential difference $\Delta V$ exists between the conductors due to the presence of the charges.

What determines how much charge is on the plates of a capacitor for a given voltage? Experiments show that the quantity of charge $Q$ on a capacitor$^1$ is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$. The proportionality constant depends on the shape and separation of the conductors.$^2$ We can write this relationship as $Q = C \Delta V$ if we define capacitance as follows:

The capacitance $C$ of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C = \frac{Q}{\Delta V} \tag{26.1}$$

$^1$ Although the total charge on the capacitor is zero (because there is as much excess positive charge on one conductor as there is excess negative charge on the other), it is common practice to refer to the magnitude of the charge on either conductor as “the charge on the capacitor.”

$^2$ The proportionality between $\Delta V$ and $Q$ can be proved from Coulomb’s law or by experiment.
Note that by definition capacitance is always a positive quantity. Furthermore, the charge \( Q \) and the potential difference \( \Delta V \) are always expressed in Equation 26.1 as positive quantities. Because the potential difference increases linearly with the stored charge, the ratio \( Q/\Delta V \) is constant for a given capacitor. Therefore, capacitance is a measure of a capacitor’s ability to store charge. Because positive and negative charges are separated in the system of two conductors in a capacitor, there is electric potential energy stored in the system.

From Equation 26.1, we see that capacitance has SI units of coulombs per volt. The SI unit of capacitance is the farad (F), which was named in honor of Michael Faraday:

\[
1 \text{ F} = 1 \text{ C/V}
\]

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads \((10^{-6} \text{ F})\) to picofarads \((10^{-12} \text{ F})\). We shall use the symbol \( \mu \text{F} \) to represent microfarads. To avoid the use of Greek letters, in practice, physical capacitors often are labeled “\( \mu \text{F} \)” for microfarads and “\( \text{mF} \)” for micromicrofarads or, equivalently, “\( \text{pF} \)” for picofarads.

Let us consider a capacitor formed from a pair of parallel plates, as shown in Figure 26.2. Each plate is connected to one terminal of a battery, which acts as a source of potential difference. If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires when the connections are made. Let us focus on the plate connected to the negative terminal of the battery. The electric field applies a force on electrons in the wire just outside this plate; this force causes the electrons to move onto the plate. This movement continues until the plate, the wire, and the terminal are all at the same electric potential. Once this equilibrium point is attained, a potential difference no longer exists between the terminal and the plate, and as a result no electric field is present in the wire, and the movement of electrons stops. The plate now carries a negative charge. A similar process occurs at the other capacitor plate, with electrons moving from the plate to the wire, leaving the plate positively charged. In this final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.

Suppose that we have a capacitor rated at 4 \( \text{pF} \). This rating means that the capacitor can store 4 \( \text{pC} \) of charge for each volt of potential difference between the two conductors. If a 9-V battery is connected across this capacitor, one of the conductors ends up with a net charge of \(-36 \text{ pC}\) and the other ends up with a net charge of \(+36 \text{ pC}\).

Quick Quiz 26.1
A capacitor stores charge \( Q \) at a potential difference \( \Delta V \). If the voltage applied by a battery to the capacitor is doubled to \( 2\Delta V \), (a) the capacitance falls to half its initial value and the charge remains the same (b) the capacitance and the charge both fall to half their initial values (c) the capacitance and the charge both double (d) the capacitance remains the same and the charge doubles.

### 26.2 Calculating Capacitance

We can derive an expression for the capacitance of a pair of oppositely charged conductors in the following manner: assume a charge of magnitude \( Q \) and calculate the potential difference using the techniques described in the preceding chapter. We then use the expression \( C = Q/\Delta V \) to evaluate the capacitance. As we might expect, we can perform this calculation relatively easily if the geometry of the capacitor is simple.

#### PITFALL PREVENTION

**26.2 Potential Difference is \( \Delta V \), not \( V \)**

We use the symbol \( \Delta V \) for the potential difference across a circuit element or a device because this is consistent with our definition of potential difference and with the meaning of the delta sign. It is a common, but confusing, practice to use the symbol \( V \) without the delta sign for a potential difference. Keep this in mind if you consult other texts.

![Figure 26.2](image-url)
While the most common situation is that of two conductors, a single conductor also has a capacitance. For example, imagine a spherical charged conductor. The electric field lines around this conductor are exactly the same as if there were a conducting shell of infinite radius, concentric with the sphere and carrying a charge of the same magnitude but opposite sign. Thus, we can identify the imaginary shell as the second conductor of a two-conductor capacitor. We now calculate the capacitance for this situation. The electric potential of the sphere of radius $R$ is simply $k_e Q / R$, and setting $V = 0$ for the infinitely large shell, we have

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\varepsilon_0 R$$  \hspace{1cm} (26.2)$$

This expression shows that the capacitance of an isolated charged sphere is proportional to its radius and is independent of both the charge on the sphere and the potential difference.

The capacitance of a pair of conductors depends on the geometry of the conductors. Let us illustrate this with three familiar geometries, namely, parallel plates, concentric cylinders, and concentric spheres. In these examples, we assume that the charged conductors are separated by a vacuum. The effect of a dielectric material placed between the conductors is treated in Section 26.5.

### Parallel-Plate Capacitors

Two parallel metallic plates of equal area $A$ are separated by a distance $d$, as shown in Figure 26.2. One plate carries a charge $Q$, and the other carries a charge $-Q$. Let us consider how the geometry of these conductors influences the capacity of the combination to store charge. Recall that charges of the same sign repel one another. As a capacitor is being charged by a battery, electrons flow into the negative plate and out of the positive plate. If the capacitor plates are large, the accumulated charges are able to distribute themselves over a substantial area, and the amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Thus, we expect the capacitance to be proportional to the plate area $A$.

Now let us consider the region that separates the plates. If the battery has a constant potential difference between its terminals, then the electric field between the plates must increase as $d$ is decreased. Let us imagine that we move the plates closer together and consider the situation before any charges have had a chance to move in response to this change. Because no charges have moved, the electric field between the plates has the same value but extends over a shorter distance. Thus, the magnitude of the potential difference between the plates $\Delta V = Ed$ (Eq. 25.6) is now smaller. The difference between this new capacitor voltage and the terminal voltage of the battery now exists as a potential difference across the wires connecting the battery to the capacitor. This potential difference results in the electric field in the wires that drives more charge onto the plates, increasing the potential difference between the plates. When the potential difference between the plates again matches that of the battery, the potential difference across the wires falls back to zero, and the flow of charge stops. Thus, moving the plates closer together causes the charge on the capacitor to increase. If $d$ is increased, the charge decreases. As a result, we expect the capacitance of the pair of plates to be inversely proportional to $d$.

We can verify these physical arguments with the following derivation. The surface charge density on either plate is $\sigma = Q / A$. If the plates are very close together (in comparison with their length and width), we can assume that the electric field is uniform between the plates and is zero elsewhere. According to the What If? feature in Example 24.8, the value of the electric field between
the plates is

\[ E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \]

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals \( Ed \) (see Eq. 25.6); therefore,

\[ \Delta V = Ed = \frac{Qd}{\epsilon_0 A} \]

Substituting this result into Equation 26.1, we find that the capacitance is

\[ C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A} \]

\[ C = \frac{\epsilon_0 A}{d} \]  \hspace{1cm} (26.3)  \textbf{Capacitance of parallel plates}

That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation, just as we expected from our conceptual argument.

A careful inspection of the electric field lines for a parallel-plate capacitor reveals that the field is uniform in the central region between the plates, as shown in Figure 26.3a. However, the field is nonuniform at the edges of the plates. Figure 26.3b is a photograph of the electric field pattern of a parallel-plate capacitor. Note the nonuniform nature of the electric field at the ends of the plates. Such end effects can be neglected if the plate separation is small compared with the length of the plates.

Figure 26.4 shows a battery connected to a single parallel-plate capacitor with a switch in the circuit. Let us identify the circuit as a system. When the switch is closed, the battery establishes an electric field in the wires and charges flow between the wires and the capacitor. As this occurs, there is a transformation of energy within the system. Before the switch is closed, energy is stored as chemical energy in the battery. This energy is transformed during the chemical reaction that occurs within the battery when it is operating in an electric circuit. When the switch is closed, some of the chemical energy in the battery is converted to electric potential energy related to the separation of positive and negative charges on the plates. As a result, we can describe a capacitor as a device that stores energy as well as charge. We will explore this energy storage in more detail in Section 26.4.

![Figure 26.3](https://example.com/fig26.3.png)

**Figure 26.3** (a) The electric field between the plates of a parallel-plate capacitor is uniform near the center but nonuniform near the edges. (b) Electric field pattern of two oppositely charged conducting parallel plates. Small pieces of thread on an oil surface align with the electric field.
Example 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor with air between the plates has an area \( A = 2.00 \times 10^{-4} \text{ m}^2 \) and a plate separation \( d = 1.00 \text{ mm} \). Find its capacitance.

**Solution** From Equation 26.3, we find that

\[
C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.00 \times 10^{-4} \text{ m}^2)}{1.00 \times 10^{-3} \text{ m}}
\]

\[
= 1.77 \times 10^{-12} \text{ F} = 1.77 \text{ pF}
\]
Cylindrical and Spherical Capacitors

From the definition of capacitance, we can, in principle, find the capacitance of any geometric arrangement of conductors. The following examples demonstrate the use of this definition to calculate the capacitance of the other familiar geometries that we mentioned: cylinders and spheres.

**Example 26.2 The Cylindrical Capacitor**

A solid cylindrical conductor of radius \( a \) and charge \( Q \) is coaxial with a cylindrical shell of negligible thickness, radius \( b > a \), and charge \( -Q \) (Fig. 26.6a). Find the capacitance of this cylindrical capacitor if its length is \( \ell \).

**Solution** It is difficult to apply physical arguments to this configuration, although we can reasonably expect the capacitance to be proportional to the cylinder length \( \ell \) for the same reason that parallel-plate capacitance is proportional to plate area: stored charges have more room in which to be distributed. If we assume that \( \ell \) is much greater than \( a \) and \( b \), we can neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 26.6b). We must first calculate the potential difference between the two cylinders, which is given in general by

\[
V_b - V_a = - \int_a^b \mathbf{E} \cdot d\mathbf{s}
\]

where \( \mathbf{E} \) is the electric field in the region between the cylinders. In Chapter 24, we showed using Gauss’s law that the magnitude of the electric field of a cylindrical charge distribution having linear charge density \( \lambda \) is \( E = 2k_e \lambda / r \) (Eq. 24.7). The same result applies here because, according to Gauss’s law, the charge on the outer cylinder does not contribute to the electric field inside it. Using this result and noting from Figure 26.6b that \( \mathbf{E} \) is along \( r \), we find that

\[
V_b - V_a = - \int_a^b E_r dr = -2k_e \lambda \int_a^b \frac{dr}{r} = -2k_e \lambda \ln \left( \frac{b}{a} \right)
\]

Substituting this result into Equation 26.1 and using the fact that \( \lambda = Q/\ell \), we obtain

\[
C = \frac{Q}{\Delta V} = \frac{Q}{(2k_e Q/\ell) \ln(b/a)} = \frac{\ell}{2k_e \ln(b/a)}
\]

where \( \Delta V \) is the magnitude of the potential difference between the cylinders, given by \( \Delta V = |V_a - V_b| = 2k_e \lambda \ln(b/a) \), a positive quantity. As predicted, the capacitance is proportional to the length of the cylinders. As we might expect, the capacitance also depends on the radii of the two cylindrical conductors. From Equation 26.4, we see that the capacitance per unit length of a combination of concentric cylindrical conductors is

\[
\frac{C}{\ell} = \frac{1}{2k_e \ln(b/a)}
\]

An example of this type of geometric arrangement is a coaxial cable, which consists of two concentric cylindrical conductors separated by an insulator. You are likely to have a coaxial cable attached to your television set or VCR if you are a subscriber to cable television. The cable carries electrical signals in the inner and outer conductors. Such a geometry is especially useful for shielding the signals from any possible external influences.

**What If?** Suppose \( b = 2.00a \) for the cylindrical capacitor. We would like to increase the capacitance, and we can do so by choosing to increase \( \ell \) by 10% or by increasing \( a \) by 10%. Which choice is more effective at increasing the capacitance?

**Answer** According to Equation 26.4, \( C \) is proportional to \( \ell \), so increasing \( \ell \) by 10% results in a 10% increase in \( C \). For the result of the change in \( a \), let us first evaluate \( C \) for \( b = 2.00a \):

\[
C = \frac{\ell}{2k_e \ln(b/a)} = \frac{\ell}{2k_e \ln(2.00)} = \frac{\ell}{2k_e (0.693)} = 0.721 \frac{\ell}{k_e}
\]

**Figure 26.6** (Example 26.2) (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius \( a \) and length \( \ell \) surrounded by a coaxial cylindrical shell of radius \( b \). (b) End view. The electric field lines are radial. The dashed line represents the end of the cylindrical gaussian surface of radius \( r \) and length \( \ell \).
Now, for a 10% increase in \( a \), the new value is \( a' = 1.10a \), so

\[
C' = \frac{\ell}{2k_r \ln(b/a')} = \frac{\ell}{2k_r \ln(2.00/1.10)} = \frac{\ell}{2k_r (0.598)} = 0.836 \frac{\ell}{k_r}
\]

The ratio of the new and old capacitances is

\[
\frac{C'}{C} = \frac{0.836 \ell/k_r}{0.721 \ell/k_r} = 1.16
\]

Corresponding to a 16% increase in capacitance. Thus, it is more effective to increase \( a \) than to increase \( \ell \).

Note two more extensions of this problem. First, the advantage goes to increasing \( a \) only for a range of relationships between \( a \) and \( b \). It is a valuable exercise to show that if \( b > 2.85a \), increasing \( \ell \) by 10% is more effective than increasing \( a \) (Problem 77). Second, if we increase \( b \), we reduce the capacitance, so we would need to decrease \( b \) to increase the capacitance. Increasing \( a \) and decreasing \( b \) both have the effect of bringing the plates closer together, which increases the capacitance.

**Example 26.3 The Spherical Capacitor**

A spherical capacitor consists of a spherical conducting shell of radius \( b \) and charge \( -Q \) concentric with a smaller conducting sphere of radius \( a \) and charge \( Q \) (Fig. 26.7). Find the capacitance of this device.

**Solution** As we showed in Chapter 24, the field outside a spherically symmetric charge distribution is radial and given by the expression \( k_r Q/r^2 \). In this case, this result applies to the field between the spheres \( (a < r < b) \). From Gauss’s law we see that only the inner sphere contributes to this field. Thus, the potential difference between the spheres is

\[
V_b - V_a = -\int_a^b E_r \, dr = -k_r Q \int_a^b \frac{dr}{r^2} = k_r Q \left[ \frac{1}{r} \right]_a^b
\]

\[
= k_r Q \left( \frac{1}{b} - \frac{1}{a} \right)
\]

The magnitude of the potential difference is

\[
\Delta V = |V_b - V_a| = k_r Q \frac{(b - a)}{ab}
\]

Substituting this value for \( \Delta V \) into Equation 26.1, we obtain

\[
C = \frac{Q}{\Delta V} = \frac{ab}{k_r(b - a)} \tag{26.6}
\]

**What if?** What if the radius \( b \) of the outer sphere approaches infinity? What does the capacitance become?

**Answer** In Equation 26.6, let \( b \to \infty \):

\[
C = \lim_{b \to \infty} \frac{ab}{k_r(b - a)} = \frac{ab}{k_r} = \frac{a}{k_r} = 4\pi\varepsilon_0 a
\]

Note that this is the same expression as Equation 26.2, the capacitance of an isolated spherical conductor.

### 26.3 Combinations of Capacitors

Two or more capacitors often are combined in electric circuits. We can calculate the equivalent capacitance of certain combinations using methods described in this section. Throughout this section, we assume that the capacitors to be combined are initially uncharged.

In studying electric circuits, we use a simplified pictorial representation called a **circuit diagram**. Such a diagram uses circuit symbols to represent various circuit elements. The circuit symbols are connected by straight lines that represent the wires between the circuit elements. The circuit symbols for capacitors and batteries, as well as the color codes used for them in this text, are given in Figure 26.8. The symbol for the capacitor reflects the geometry of the most common model for a capacitor—a pair of parallel plates. The positive terminal of the battery is at the higher potential and is represented in the circuit symbol by the longer line.

---

**Figure 26.7** (Example 26.3) A spherical capacitor consists of an inner sphere of radius \( a \) surrounded by a concentric spherical shell of radius \( b \). The electric field between the spheres is directed radially outward when the inner sphere is positively charged.

**Figure 26.8** Circuit symbols for capacitors, batteries, and switches. Note that capacitors are in blue and batteries and switches are in red.
**Parallel Combination**

Two capacitors connected as shown in Figure 26.9a are known as a *parallel combination* of capacitors. Figure 26.9b shows a circuit diagram for this combination of capacitors. The left plates of the capacitors are connected by a conducting wire to the positive terminal of the battery and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and are therefore both at the same potential as the negative terminal. Thus, the **individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.**

In a circuit such as that shown in Figure 26.9, the voltage applied across the combination is the terminal voltage of the battery. Situations can occur in which the parallel combination is in a circuit with other circuit elements; in such situations, we must determine the potential difference across the combination by analyzing the entire circuit.

When the capacitors are first connected in the circuit shown in Figure 26.9, electrons are transferred between the wires and the plates; this transfer leaves the left plates positively charged and the right plates negatively charged. The flow of charge ceases when the voltage across the capacitors is equal to that across the battery terminals. The capacitors reach their maximum charge when the flow of charge ceases. Let us call the maximum charges on the two capacitors $Q_1$ and $Q_2$. The total charge $Q$ stored by the two capacitors is

$$Q = Q_1 + Q_2 \quad (26.7)$$

That is, **the total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors.** Because the voltages across the capacitors are the same, the charges that they carry are

$$Q_1 = C_1 \Delta V \quad Q_2 = C_2 \Delta V$$

Suppose that we wish to replace these two capacitors by one *equivalent capacitor* having a capacitance $C_{eq}$ as in Figure 26.9c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two

---

**Active Figure 26.9** (a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is $\Delta V$. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$. 

---

At the Active Figures link at http://www.pse6.com, you can adjust the battery voltage and the individual capacitances to see the resulting charges and voltages on the capacitors. You can combine up to four capacitors in parallel.
individual capacitors. That is, the equivalent capacitor must store $Q$ units of charge when connected to the battery. We can see from Figure 26.9c that the voltage across the equivalent capacitor also is $\Delta V$ because the equivalent capacitor is connected directly across the battery terminals. Thus, for the equivalent capacitor,

$$Q = C_{eq} \Delta V$$

Substituting these three relationships for charge into Equation 26.7, we have

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$
$$C_{eq} = C_1 + C_2 \quad \text{(parallel combination)}$$

If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(parallel combination)} \quad (26.8)$$

Thus, the equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances and is greater than any of the individual capacitances. This makes sense because we are essentially combining the areas of all the capacitor plates when we connect them with conducting wire, and capacitance of parallel plates is proportional to area (Eq. 26.3).

**Series Combination**

Two capacitors connected as shown in Figure 26.10a and the equivalent circuit diagram in Figure 26.10b are known as a series combination of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else; hence, they form an isolated conductor that is initially uncharged and must continue to have zero net charge. To analyze this combination, let us begin by considering the uncharged capacitors and follow what happens just after a battery is connected to the circuit. When the battery is connected, electrons are transferred out of the left plate of $C_1$ and into the right plate of $C_2$. As this negative charge accumulates on the right plate of $C_2$, an equivalent amount of negative charge is forced off the left plate of $C_2$, and this left plate therefore has an excess positive charge. The negative charge leaving

![Diagram of capacitors in parallel and series connections.](image-url)
the left plate of $C_2$ causes negative charges to accumulate on the right plate of $C_1$. As a result, all the right plates end up with a charge $-Q$, and all the left plates end up with a charge $+Q$. Thus, the charges on capacitors connected in series are the same.

From Figure 26.10a, we see that the voltage $\Delta V$ across the battery terminals is split between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2$$  \hspace{1cm} (26.9)

where $\Delta V_1$ and $\Delta V_2$ are the potential differences across capacitors $C_1$ and $C_2$, respectively. In general, the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

Suppose that the equivalent single capacitor in Figure 26.10c has the same effect on the circuit as the series combination when it is connected to the battery. After it is fully charged, the equivalent capacitor must have a charge of $-Q$ on its right plate and a charge of $+Q$ on its left plate. Applying the definition of capacitance to the circuit in Figure 26.10c, we have

$$\Delta V = \frac{Q}{C_{eq}}$$

Because we can apply the expression $Q = C \Delta V$ to each capacitor shown in Figure 26.10b, the potential differences across them are

$$\Delta V_1 = \frac{Q}{C_1} \quad \Delta V_2 = \frac{Q}{C_2}$$

Substituting these expressions into Equation 26.9, we have

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling $Q$, we arrive at the relationship

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$  \hspace{1cm} (series combination)

When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$  \hspace{1cm} (series combination)  \hspace{1cm} (26.10)

This shows that the inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.

**Quick Quiz 26.3** Two capacitors are identical. They can be connected in series or in parallel. If you want the smallest equivalent capacitance for the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same capacitance?

**Quick Quiz 26.4** Consider the two capacitors in Quick Quiz 26.3 again. Each capacitor is charged to a voltage of 10 V. If you want the largest combined potential difference across the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same potential difference?
**Example 26.4 Equivalent Capacitance**

Find the equivalent capacitance between \( a \) and \( b \) for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.

**Solution** Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The 1.0-\( \mu \)F and 3.0-\( \mu \)F capacitors are in parallel and combine according to the expression

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0 \ \mu F} + \frac{1}{4.0 \ \mu F} = \frac{1}{2.0 \ \mu F}
\]

Thus, \( C_{eq} = 2.0 \ \mu F \). The 2.0-\( \mu \)F and 6.0-\( \mu \)F capacitors also are in parallel and have an equivalent capacitance of 4.0 \( \mu \)F. Thus, the upper branch in Figure 26.11b consists of two 4.0-\( \mu \)F capacitors in series, which combine as follows:

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0 \ \mu F} + \frac{1}{4.0 \ \mu F} = \frac{1}{2.0 \ \mu F}
\]

**Figure 26.11** (Example 26.4) To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b), (c), and (d), using the series and parallel rules described in the text.
26.4 Energy Stored in a Charged Capacitor

Almost everyone who works with electronic equipment has at some time verified that a capacitor can store energy. If the plates of a charged capacitor are connected by a conductor, such as a wire, charge moves between each plate and its connecting wire until the capacitor is uncharged. The discharge can often be observed as a visible spark. If you should accidentally touch the opposite plates of a charged capacitor, your fingers act as a pathway for discharge, and the result is an electric shock. The degree of shock you receive depends on the capacitance and on the voltage applied to the capacitor. Such a shock could be fatal if high voltages are present, such as in the power supply of a television set. Because the charges can be stored in a capacitor even when the set is turned off, unplugging the television does not make it safe to open the case and touch the components inside.

To calculate the energy stored in the capacitor, we shall assume a charging process that is different from the actual process described in Section 26.1 but which gives the same final result. We can make this assumption because the energy in the final configuration does not depend on the actual charge-transfer process. We imagine that the charge is transferred mechanically through the space between the plates. We reach in and grab a small amount of positive charge on the plate connected to the negative terminal and apply a force that causes this positive charge to move over to the plate connected to the positive terminal. Thus, we do work on the charge as we transfer it from one plate to the other. At first, no work is required to transfer a small amount of charge $dq$ from one plate to the other. However, once this charge has been transferred, a small potential difference exists between the plates. Therefore, work must be done to move additional charge through this potential difference. As more and more charge is transferred from one plate to the other, the potential difference increases in proportion, and more work is required.

Suppose that $q$ is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. From Section 25.2, we know that the work necessary to transfer an increment of charge $dq$ from the plate carrying charge $-q$ to the plate carrying charge $q$ (which is at the higher electric potential) is

$$dW = \Delta V dq = \frac{q}{C} dq$$

This is illustrated in Figure 26.12. The total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$

The work done in charging the capacitor appears as electric potential energy $U$ stored in the capacitor. Using Equation 26.1, we can express the potential energy stored in a charged capacitor in the following forms:

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Figure 26.12 A plot of potential difference versus charge for a capacitor is a straight line having a slope $1/C$. The work required to move charge $dq$ through the potential difference $\Delta V$ existing at the time across the capacitor plates is given approximately by the area of the shaded rectangle. The total work required to charge the capacitor to a final charge $Q$ is the triangular area under the straight line, $W = \frac{1}{2} Q \Delta V$. (Don’t forget that $1 \text{ V} = 1 \text{ J/C}$, hence, the unit for the triangular area is the joule.)

This result applies to any capacitor, regardless of its geometry. We see that for a given capacitance, the stored energy increases as the charge increases and as the potential difference increases. In practice, there is a limit to the maximum energy (or charge) that can be stored because, at a sufficiently great value of $\Delta V$, discharge ultimately occurs between the plates. For this reason, capacitors are usually labeled with a maximum operating voltage.

---

3 We shall use lowercase $q$ for the time-varying charge on the capacitor while it is charging, to distinguish it from uppercase $Q$, which is the total charge on the capacitor after it is completely charged.
We can consider the energy stored in a capacitor as being stored in the electric field created between the plates as the capacitor is charged. This description is reasonable because the electric field is proportional to the charge on the capacitor. For a parallel-plate capacitor, the potential difference is related to the electric field through the relationship \( \Delta V = Ed \). Furthermore, its capacitance is \( C = \epsilon_0 A/d \) (Eq. 26.3). Substituting these expressions into Equation 26.11, we obtain

\[
U = \frac{1}{2} \frac{\epsilon_0 A}{d} (E^2 d^2) = \frac{1}{2} (\epsilon_0 A) E^2
\]

(26.12)

Because the volume occupied by the electric field is \( Ad \), the energy per unit volume \( u_E = U/Ad \), known as the energy density, is

\[
u_E = \frac{1}{2} \epsilon_0 E^2
\]

(26.13)

Although Equation 26.13 was derived for a parallel-plate capacitor, the expression is generally valid, regardless of the source of the electric field. That is, the energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

**Quick Quiz 26.5** You have three capacitors and a battery. In which of the following combinations of the three capacitors will the maximum possible energy be stored when the combination is attached to the battery? (a) series (b) parallel (c) Both combinations will store the same amount of energy.

**Quick Quiz 26.6** You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates apart to a larger separation, do the following quantities increase, decrease, or stay the same? (a) \( C \); (b) \( Q \); (c) \( E \) between the plates; (d) \( \Delta V \); (e) energy stored in the capacitor.

**Quick Quiz 26.7** Repeat Quick Quiz 26.6, but this time answer the questions for the situation in which the battery remains connected to the capacitor while you pull the plates apart.

---

**Example 26.5  Rewiring Two Charged Capacitors**

Two capacitors \( C_1 \) and \( C_2 \) (where \( C_1 > C_2 \)) are charged to the same initial potential difference \( \Delta V_i \). The charged capacitors are removed from the battery, and their plates are connected with opposite polarity as in Figure 26.13a. The switches \( S_1 \) and \( S_2 \) are then closed, as in Figure 26.13b.

(A) Find the final potential difference \( \Delta V_f \) between \( a \) and \( b \) after the switches are closed.

**Solution** Figure 26.13 helps us conceptualize the initial and final configurations of the system. In Figure 26.13b, it might appear as if the capacitors are connected in parallel, but there is no battery in this circuit that is applying a voltage across the combination. Thus, we cannot categorize this as a problem in which capacitors are connected in parallel. We can categorize this as a problem involving an isolated system for electric charge—the left-hand plates of the capacitors form an isolated system because they are not connected to the right-hand plates by conductors. To analyze
the problem, note that the charges on the left-hand plates before the switches are closed are

\[ Q_{1i} = C_1 \Delta V_i \quad \text{and} \quad Q_{2i} = -C_2 \Delta V_i \]

The negative sign for \( Q_{2i} \) is necessary because the charge on the left plate of capacitor \( C_2 \) is negative. The total charge \( Q \) in the system is

\[ Q = Q_{1i} + Q_{2i} = (C_1 - C_2) \Delta V_i \]

(1)

After the switches are closed, the total charge \( Q \) in the system remains the same but the charges on the individual capacitors change to new values \( Q_{1f} \) and \( Q_{2f} \). Because the system is isolated,

\[ Q = Q_{1f} + Q_{2f} \]

(2)

The charges redistribute until the potential difference is the same across both capacitors, \( \Delta V_f \). To satisfy this requirement, the charges on the capacitors after the switches are closed are

\[ Q_{1f} = C_1 \Delta V_f \quad \text{and} \quad Q_{2f} = C_2 \Delta V_f \]

Dividing the first equation by the second, we have

\[ Q_{1f} = \frac{C_1}{C_2} Q_{2f} \]

(3)

Combining Equations (2) and (3), we obtain

\[ Q = Q_{1f} + Q_{2f} = \frac{C_1}{C_2} Q_{2f} + Q_{2f} = Q_{2f} \left(1 + \frac{C_1}{C_2}\right) \]

(4)

Using Equations (3) and (4) to find \( Q_{1f} \) in terms of \( Q \), we have

\[ Q_{1f} = \frac{C_1}{C_2} Q_{2f} = \frac{C_2}{C_2} Q \left( \frac{C_2}{C_1 + C_2} \right) \]

(5)

\[ = Q \left( \frac{C_1}{C_1 + C_2} \right) \]

Finally, using Equation 26.1 to find the voltage across each capacitor, we find that

\[ \Delta V_{1f} = \frac{Q_{1f}}{C_1} = \frac{Q \left( \frac{C_1}{C_1 + C_2} \right)}{C_1} = \frac{Q}{C_1 + C_2} \]

(6)

\[ \Delta V_{2f} = \frac{Q_{2f}}{C_2} = \frac{Q \left( \frac{C_2}{C_1 + C_2} \right)}{C_2} = \frac{Q}{C_1 + C_2} \]

(7)

As noted earlier, \( \Delta V_{1f} = \Delta V_{2f} = \Delta V_f \).

To express \( \Delta V_f \) in terms of the given quantities \( C_1, C_2, \) and \( \Delta V_i \), we substitute the value of \( Q \) from Equation (1) into either Equation (6) or (7) to obtain

\[ \Delta V_f = \left( \frac{C_1 - C_2}{C_1 + C_2} \right) \Delta V_i \]

(8)

**Answer** The equal-magnitude charges on the two capacitors should simply cancel each other and the capacitors will be uncharged afterward.

Let us test our results to see if this is the case mathematically. In Equation (1), because the charges are of equal magnitude and opposite sign, we see that \( Q = 0 \). Thus, Equations (4) and (5) show us that \( Q_{1f} = Q_{2f} = 0 \), consistent with our prediction. Furthermore, Equations (6) and (7) show us that \( \Delta V_{1f} = \Delta V_{2f} = 0 \), which is consistent with uncharged capacitors. Finally, if \( C_1 = C_2 \), Equation (8) shows us that \( \Delta V_f = 0 \), which is also consistent with uncharged capacitors.

---

**Solution** Before the switches are closed, the total energy stored in the capacitors is

\[ U_i = \frac{1}{2} C_1 (\Delta V_i)^2 + \frac{1}{2} C_2 (\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2) (\Delta V_i)^2 \]

After the switches are closed, the total energy stored in the capacitors is

\[ U_f = \frac{1}{2} C_1 (\Delta V_f)^2 + \frac{1}{2} C_2 (\Delta V_f)^2 = \frac{1}{2}(C_1 + C_2) (\Delta V_f)^2 \]

Using the results of part (A), we can express this as

\[ U_f = \left( \frac{\frac{1}{2} (C_1 - C_2)^2 (\Delta V_i)^2}{(C_1 + C_2)} \right) \]

Therefore, the ratio of the final energy stored to the initial energy stored is

\[ \frac{U_f}{U_i} = \frac{\frac{1}{2} (C_1 - C_2)^2 (\Delta V_i)^2}{(C_1 + C_2)^2} \]

To finalize this problem, note that this ratio is less than unity, indicating that the final energy is less than the initial energy. At first, you might think that the law of energy conservation has been violated, but this is not the case. The “missing” energy is transferred out of the system of the capacitors by the mechanism of electromagnetic waves, as we shall see in Chapter 34.

**What If?** What if the two capacitors have the same capacitance? What would we expect to happen when the switches are closed?
One device in which capacitors have an important role is the defibrillator (Fig. 26.14). Up to 360 J is stored in the electric field of a large capacitor in a defibrillator when it is fully charged. The defibrillator can deliver all this energy to a patient in about 2 ms. (This is roughly equivalent to 3000 times the power delivered to a 60-W lightbulb!) Under the proper conditions, the defibrillator can be used to stop cardiac fibrillation (random contractions) in heart attack victims. When fibrillation occurs, the heart produces a rapid, irregular pattern of beats. A fast discharge of energy through the heart can return the organ to its normal beat pattern. A fast discharge of energy through the heart can return the organ to its normal beat pattern. Emergency medical teams use portable defibrillators that contain batteries capable of charging a capacitor to a high voltage. (The circuitry actually permits the capacitor to be charged to a much higher voltage than that of the battery.) The stored energy is released through the heart by conducting electrodes, called paddles, that are placed on both sides of the victim’s chest. The paramedics must wait between applications of the energy due to the time necessary for the capacitors to become fully charged. In this case and others (e.g., camera flash units and lasers used for fusion experiments), capacitors serve as energy reservoirs which can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse.

A camera’s flash unit also uses a capacitor, although the total amount of energy stored is much less than that stored in a defibrillator. After the flash unit’s capacitor is charged, tripping the camera’s shutter causes the stored energy to be sent through a special lightbulb that briefly illuminates the subject being photographed.

26.5 Capacitors with Dielectrics

A dielectric is a nonconducting material, such as rubber, glass, or waxed paper. When a dielectric is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor $\kappa$, which is called the dielectric constant of the material. The dielectric constant varies from one material to another. In this section, we analyze this change in capacitance in terms of electrical parameters such as electric charge, electric field, and potential difference; in Section 26.7, we shall discuss the microscopic origin of these changes.
If the dielectric is introduced while the potential difference is held constant by a battery, the charge increases to a value \( Q = \frac{Q_0}{\kappa} \). The additional charge comes from the wires attached to the capacitor, and the capacitance again increases by the factor \( \kappa \).

We can perform the following experiment to illustrate the effect of a dielectric in a capacitor. Consider a parallel-plate capacitor that without a dielectric has a charge \( Q_0 \) and a capacitance \( C_0 \). The potential difference across the capacitor is \( V_0 = \frac{Q_0}{C_0} \). Figure 26.15a illustrates this situation. The potential difference is measured by a voltmeter, which we shall study in greater detail in Chapter 28. Note that no battery is shown in the figure; also, we must assume that no charge can flow through an ideal voltmeter. Hence, there is no path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates, as in Figure 26.15b, the voltmeter indicates that the voltage between the plates decreases to a value \( V \). The voltages with and without the dielectric are related by the factor \( \kappa \) as follows:

\[
\Delta V = \frac{\Delta V_0}{\kappa}
\]

Because \( \Delta V < \Delta V_0 \), we see that \( \kappa > 1 \).

Because the charge \( Q_0 \) on the capacitor does not change, we conclude that the capacitance must change to the value

\[
C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}
\]

\[
C = \kappa C_0 \tag{26.14}
\]

That is, the capacitance increases by the factor \( \kappa \) when the dielectric completely fills the region between the plates. For a parallel-plate capacitor, where \( C_0 = \varepsilon_0 A/d \) (Eq. 26.3), we can express the capacitance when the capacitor is filled with a dielectric as

\[
C = \kappa \frac{\varepsilon_0 A}{d} \tag{26.15}
\]

From Equations 26.3 and 26.15, it would appear that we could make the capacitance very large by decreasing \( d \), the distance between the plates. In practice, the lowest value of \( d \) is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation \( d \), the maximum voltage that can be applied to a capacitor without causing a discharge depends on the

\footnote{If the dielectric is introduced while the potential difference is held constant by a battery, the charge increases to a value \( Q = \kappa Q_0 \). The additional charge comes from the wires attached to the capacitor, and the capacitance again increases by the factor \( \kappa \).}
dielectric strength (maximum electric field) of the dielectric. If the magnitude of the electric field in the dielectric exceeds the dielectric strength, then the insulating properties break down and the dielectric begins to conduct. Figure 26.16 shows the effect of exceeding the dielectric strength of air. Sparks appear between the two wires, due to ionization of atoms and recombination with electrons in the air, similar to the process that produced corona discharge in Section 25.6.

Physical capacitors have a specification called by a variety of names, including working voltage, breakdown voltage, and rated voltage. This parameter represents the largest voltage that can be applied to the capacitor without exceeding the dielectric strength of the dielectric material in the capacitor. Consequently, when selecting a capacitor for a given application, you must consider the capacitance of the device along with the expected voltage across the capacitor in the circuit, making sure that the expected voltage will be smaller than the rated voltage of the capacitor. You can see the rated voltage on several of the capacitors in the opening photograph for this chapter.

Insulating materials have values of $\kappa$ greater than unity and dielectric strengths greater than that of air, as Table 26.1 indicates. Thus, we see that a dielectric provides the following advantages:

- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing $d$ and increasing $C$.

### Types of Capacitors

Commercial capacitors are often made from metallic foil interlaced with thin sheets of either paraffin-impregnated paper or Mylar as the dielectric material. These alternate layers of metallic foil and dielectric are rolled into a cylinder to form a small package (Fig. 26.17a). High-voltage capacitors commonly consist of a number of interwoven

### Table 26.1

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant $\kappa$</th>
<th>Dielectric Strength$^a$ (10$^6$ V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (dry)</td>
<td>1.000 59</td>
<td>3</td>
</tr>
<tr>
<td>Bakelite</td>
<td>4.9</td>
<td>24</td>
</tr>
<tr>
<td>Fused quartz</td>
<td>3.78</td>
<td>8</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.2</td>
<td>7</td>
</tr>
<tr>
<td>Neoprene rubber</td>
<td>6.7</td>
<td>12</td>
</tr>
<tr>
<td>Nylon</td>
<td>3.4</td>
<td>14</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
<td>16</td>
</tr>
<tr>
<td>Paraffin-impregnated paper</td>
<td>3.5</td>
<td>11</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.56</td>
<td>24</td>
</tr>
<tr>
<td>Polivinyl chloride</td>
<td>3.4</td>
<td>40</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>5.6</td>
<td>14</td>
</tr>
<tr>
<td>Silicone oil</td>
<td>2.5</td>
<td>15</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>233</td>
<td>8</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>60</td>
</tr>
<tr>
<td>Vacuum</td>
<td>1,000 00</td>
<td>—</td>
</tr>
<tr>
<td>Water</td>
<td>80</td>
<td>—</td>
</tr>
</tbody>
</table>

$^a$ The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.
metallic plates immersed in silicone oil (Fig. 26.17b). Small capacitors are often constructed from ceramic materials.

Often, an *electrolytic capacitor* is used to store large amounts of charge at relatively low voltages. This device, shown in Figure 26.17c, consists of a metallic foil in contact with an *electrolyte*—a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, and this layer serves as the dielectric. Very large values of capacitance can be obtained in an electrolytic capacitor because the dielectric layer is very thin, and thus the plate separation is very small.

Electrolytic capacitors are not reversible as are many other capacitors—they have a polarity, which is indicated by positive and negative signs marked on the device. When electrolytic capacitors are used in circuits, the polarity must be aligned properly. If the polarity of the applied voltage is opposite that which is intended, the oxide layer is removed and the capacitor conducts electricity instead of storing charge.

Variable capacitors (typically 10 to 500 pF) usually consist of two interwoven sets of metallic plates, one fixed and the other movable, and contain air as the dielectric (Fig. 26.18). These types of capacitors are often used in radio tuning circuits.

**Quick Quiz 26.8** If you have ever tried to hang a picture or a mirror, you know it can be difficult to locate a wooden stud in which to anchor your nail or screw. A carpenter’s stud-finder is basically a capacitor with its plates arranged side by side instead of facing one another, as shown in Figure 26.19. When the device is moved over a stud, does the capacitance increase or decrease?

![Stud-finder diagram](image)

*Figure 26.19* (Quick Quiz 26.8) A stud-finder. (a) The materials between the plates of the capacitor are the wallboard and air. (b) When the capacitor moves across a stud in the wall, the materials between the plates are the wallboard and the wood. The change in the dielectric constant causes a signal light to illuminate.
Quick Quiz 26.9 A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities increase, decrease, or stay the same? (a) \(C\); (b) \(Q\); (c) \(E\) between the plates; (d) \(\Delta V\).

Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

(A) Find its capacitance.

Solution Because \(\kappa = 3.7\) for paper (see Table 26.1), we have

\[
C = \kappa \frac{\varepsilon_0 A}{d} = 3.7 \left( \frac{(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2)(6.0 \times 10^{-4} \text{m}^2)}{1.0 \times 10^{-3} \text{m}} \right) = 20 \times 10^{-12} \text{F} = 20 \text{pF}
\]

(B) What is the maximum charge that can be placed on the capacitor?

Solution From Table 26.1 we see that the dielectric strength of paper is \(16 \times 10^6 \text{V/m}\). Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is

\[
\Delta V_{\text{max}} = E_{\text{max}} d = (16 \times 10^6 \text{V/m})(1.0 \times 10^{-3} \text{m}) = 16 \times 10^3 \text{V}
\]

Hence, the maximum charge is

\[
Q_{\text{max}} = C \Delta V_{\text{max}} = (20 \times 10^{-12} \text{F})(16 \times 10^3 \text{V}) = 0.32 \mu \text{C}
\]

Example 26.7 Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge \(Q_0\), as shown in Figure 26.20a. The battery is then removed, and a slab of material that has a dielectric constant \(\kappa\) is inserted between the plates, as shown in Figure 26.20b. Find the energy stored in the capacitor before and after the dielectric is inserted.

\[
U_0 = \frac{Q_0^2}{2C_0}
\]

After the battery is removed and the dielectric inserted, the charge on the capacitor remains the same. Hence, the energy stored in the presence of the dielectric is

\[
U = \frac{Q_0^2}{2C}
\]

But the capacitance in the presence of the dielectric is \(C = \kappa C_0\), so \(U\) becomes

\[
U = \frac{Q_0^2}{2\kappa C_0} = \frac{U_0}{\kappa}
\]

Because \(\kappa > 1\), the final energy is less than the initial energy. We can account for the “missing” energy by noting that the dielectric, when inserted, is pulled into the device (see Section 26.7). An external agent must do negative work to keep the dielectric from accelerating. This work is simply the difference \(U - U_0\). (Alternatively, the positive work done by the system on the external agent is \(U_0 - U\).)
26.6 Electric Dipole in an Electric Field

We have discussed the effect of placing a dielectric between the plates of a capacitor. In Section 26.7, we shall describe the microscopic origin of this effect. Before we can do so, however, we need to expand upon the discussion of the electric dipole that we began in Section 23.4 (see Example 23.6). The electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure 26.21. The electric dipole moment of this configuration is defined as the vector $\mathbf{p}$ directed from $-q$ toward $+q$ along the line joining the charges and having magnitude $2aq$:

$$ p = 2aq \quad (26.16) $$

Now suppose that an electric dipole is placed in a uniform electric field $\mathbf{E}$, as shown in Figure 26.22. We identify $\mathbf{E}$ as the field external to the dipole, distinguishing it from the field due to the dipole, which we discussed in Section 23.4. The field $\mathbf{E}$ is established by some other charge distribution, and we place the dipole into this field. Let us imagine that the dipole moment makes an angle $\theta$ with the field.

The electric forces acting on the two charges are equal in magnitude ($F = qE$) and opposite in direction as shown in Figure 26.22. Thus, the net force on the dipole is zero. However, the two forces produce a net torque on the dipole; as a result, the dipole rotates in the direction that brings the dipole moment vector into greater alignment with the field. The torque due to the force on the positive charge about an axis through $O$ in Figure 26.22 has magnitude $Fa \sin \theta$, where $a \sin \theta$ is the moment arm of $F$ about $O$. This force tends to produce a clockwise rotation. The torque about $O$ on the negative charge is also of magnitude $Fa \sin \theta$; here again, the force tends to produce a clockwise rotation. Thus, the magnitude of the net torque about $O$ is

$$ \tau = 2Fa \sin \theta $$

Because $F = qE$ and $p = 2aq$, we can express $\tau$ as

$$ \tau = 2aqE \sin \theta = pE \sin \theta \quad (26.17) $$

It is convenient to express the torque in vector form as the cross product of the vectors $\mathbf{p}$ and $\mathbf{E}$:

$$ \tau = \mathbf{p} \times \mathbf{E} \quad (26.18) $$

We can determine the potential energy of the system—an electric dipole in an external electric field—as a function of the orientation of the dipole with respect to the field. To do this, we recognize that work must be done by an external agent to rotate the dipole through an angle so as to cause the dipole moment vector to become less aligned with the field. The work done is then stored as potential energy in the system. The work $dW$ required to rotate the dipole through an angle $d\theta$ is $dW = \tau d\theta$ (Eq. 10.22). Because $\tau = pE \sin \theta$ and because the work results in an increase in the potential energy $U$, we find that for a rotation from $\theta_i$ to $\theta_f$ the change in potential energy of the system is

$$ U_f - U_i = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} pE \sin \theta \ d\theta = pE \int_{\theta_i}^{\theta_f} \sin \theta \ d\theta = pE \left[ -\cos \theta \right]_{\theta_i}^{\theta_f} = pE (\cos \theta_f - \cos \theta_i) $$

The term that contains $\cos \theta_i$ is a constant that depends on the initial orientation of the dipole. It is convenient for us to choose a reference angle of $\theta_i = 90^\circ$, so that $\cos \theta_i = \cos 90^\circ = 0$. Furthermore, let us choose $U_i = 0$ at $\theta_i = 90^\circ$ as our reference of potential energy. Hence, we can express a general value of $U = U_f$ as

$$ U = -pE \cos \theta \quad (26.19) $$

We can write this expression for the potential energy of a dipole in an electric field as the dot product of the vectors $\mathbf{p}$ and $\mathbf{E}$:
Potential energy of the system of an electric dipole in an external electric field

To develop a conceptual understanding of Equation 26.19, compare this expression with the expression for the potential energy of the system of an object in the gravitational field of the Earth, \( U = mgh \) (see Chapter 8). The gravitational expression includes a parameter associated with the object we place in the field—its mass \( m \). Likewise, Equation 26.19 includes a parameter of the object in the electric field—its dipole moment \( p \). The gravitational expression includes the magnitude of the gravitational field \( g \). Similarly, Equation 26.19 includes the magnitude of the electric field \( E \).

So far, these two contributions to the potential energy expressions appear analogous. However, the final contribution is somewhat different in the two cases. In the gravitational expression, the potential energy depends on how high we lift the object, measured by \( h \). In Equation 26.19, the potential energy depends on the angle \( \theta \) through which we rotate the dipole. In both cases, we are making a change in the configuration of the system. In the gravitational case, the change involves moving an object in a translational sense, whereas in the electrical case, the change involves moving an object in a rotational sense. In both cases, however, once the change is made, the system tends to return to the original configuration when the object is released: the object of mass \( m \) falls back to the ground, and the dipole begins to rotate back toward the configuration in which it is aligned with the field. Thus, apart from the type of motion, the expressions for potential energy in these two cases are similar.

Molecules are said to be polarized when a separation exists between the average position of the negative charges and the average position of the positive charges in the molecule. In some molecules, such as water, this condition is always present—such molecules are called polar molecules. Molecules that do not possess a permanent polarization are called nonpolar molecules.

We can understand the permanent polarization of water by inspecting the geometry of the water molecule. In the water molecule, the oxygen atom is bonded to the hydrogen atoms such that an angle of 105° is formed between the two bonds (Fig. 26.23). The center of the negative charge distribution is near the oxygen atom, and the center of the positive charge distribution lies at a point midway along the line joining the hydrogen atoms (the point labeled \( \times \) in Fig. 26.23). We can model the water molecule and other polar molecules as dipoles because the average positions of the positive and negative charges act as point charges. As a result, we can apply our discussion of dipoles to the behavior of polar molecules.

Microwave ovens take advantage of the polar nature of the water molecule. When in operation, microwave ovens generate a rapidly changing electric field that causes the polar molecules to swing back and forth, absorbing energy from the field in the process. Because the jostling molecules collide with each other, the energy they absorb from the field is converted to internal energy, which corresponds to an increase in temperature of the food.

Another household scenario in which the dipole structure of water is exploited is washing with soap and water. Grease and oil are made up of nonpolar molecules, which are generally not attracted to water. Plain water is not very useful for removing this type of grime. Soap contains long molecules called surfactants. In a long molecule, the polarity characteristics of one end of the molecule can be different from those at the other end. In a surfactant molecule, one end acts like a nonpolar molecule and the other acts like a polar molecule. The nonpolar end can attach to a grease or oil molecule, and the polar end can attach to a water molecule. Thus, the soap serves as a chain, linking the dirt and water molecules together. When the water is rinsed away, the grease and oil go with it.

A symmetric molecule (Fig. 26.24a) has no permanent polarization, but polarization can be induced by placing the molecule in an electric field. A field directed to the left, as shown in Figure 26.24b, would cause the center of the positive charge distribution to shift to the left from its initial position and the center of the negative charge distribution to shift to the right. This induced polarization is the effect that predominates in most materials used as dielectrics in capacitors.
Example 26.8 The H\textsubscript{2}O Molecule

The water (H\textsubscript{2}O) molecule has an electric dipole moment of 6.3 \times 10^{-30} \text{ C} \cdot \text{m}. A sample contains 10\textsuperscript{21} water molecules, with the dipole moments all oriented in the direction of an electric field of magnitude 2.5 \times 10^5 \text{ N/C}. How much work is required to rotate the dipoles from this orientation (\(\theta = 0^\circ\)) to one in which all the moments are perpendicular to the field (\(\theta = 90^\circ\))?

Solution The work required to rotate one molecule 90\(^\circ\) is equal to the difference in potential energy between the 90\(^\circ\) orientation and the 0\(^\circ\) orientation. Using Equation 26.19, we obtain

\[
W = U_{\text{ind}} - U_0 = (-pE \cos 90^\circ) - (-pE \cos 0^\circ)
\]

\[
= pE = (6.3 \times 10^{-30} \text{ C} \cdot \text{m})(2.5 \times 10^5 \text{ N/C})
\]

\[
= 1.6 \times 10^{-21} \text{ J}
\]

Because there are 10\textsuperscript{21} molecules in the sample, the total work required is

\[
W_{\text{total}} = (10^{21})(1.6 \times 10^{-24} \text{ J}) = 1.6 \times 10^{-3} \text{ J}
\]

26.7 An Atomic Description of Dielectrics

In Section 26.5 we found that the potential difference \(\Delta V_0\) between the plates of a capacitor is reduced to \(\Delta V_0/\kappa\) when a dielectric is introduced. The potential difference is reduced because the magnitude of the electric field decreases between the plates. In particular, if \(E_0\) is the electric field without the dielectric, the field in the presence of a dielectric is

\[
E = \frac{E_0}{\kappa}
\]  

(26.21)

Let us first consider a dielectric made up of polar molecules placed in the electric field between the plates of a capacitor. The dipoles (that is, the polar molecules making up the dielectric) are randomly oriented in the absence of an electric field, as shown in Figure 26.25a. When an external field \(E_0\) due to charges on the capacitor plates is applied, a torque is exerted on the dipoles, causing them to partially align with the field, as shown in Figure 26.25b. We can now describe the dielectric as being polarized. The degree of alignment of the molecules with the electric field depends on temperature and on the magnitude of the field. In general, the alignment increases with decreasing temperature and with increasing electric field.

If the molecules of the dielectric are nonpolar, then the electric field due to the plates produces some charge separation and an induced dipole moment. These induced dipole moments tend to align with the external field, and the dielectric is polarized. Thus, we can polarize a dielectric with an external field regardless of whether the molecules are polar or nonpolar.

![Figure 26.25](image)

Figure 26.25 (a) Polar molecules are randomly oriented in the absence of an external electric field. (b) When an external electric field is applied, the molecules partially align with the field. (c) The charged edges of the dielectric can be modeled as an additional pair of parallel plates establishing an electric field \(E_{\text{ind}}\) in the direction opposite to that of \(E_0\).
With these ideas in mind, consider a slab of dielectric material placed between the plates of a capacitor so that it is in a uniform electric field $E_0$, as shown in Figure 26.25b. The electric field due to the plates is directed to the right and polarizes the dielectric. The net effect on the dielectric is the formation of an induced positive surface charge density $\sigma_{\text{ind}}$ on the right face and an equal-magnitude negative surface charge density $-\sigma_{\text{ind}}$ on the left face, as shown in Figure 26.25c. Because we can model these surface charge distributions as being due to parallel plates, the induced surface charges on the dielectric give rise to an induced electric field $E_{\text{ind}}$ in the direction opposite the external field $E_0$. Therefore, the net electric field $E$ in the dielectric has a magnitude

$$E = E_0 - E_{\text{ind}} \tag{26.22}$$

In the parallel-plate capacitor shown in Figure 26.26, the external field $E_0$ is related to the charge density $\sigma$ on the plates through the relationship $E_0 = \sigma/\epsilon_0$. The induced electric field in the dielectric is related to the induced charge density $\sigma_{\text{ind}}$ through the relationship $E_{\text{ind}} = \sigma_{\text{ind}}/\epsilon_0$. Because $E = E_0/\kappa = \sigma/\kappa\epsilon_0$, substitution into Equation 26.22 gives

$$\frac{\sigma}{\kappa\epsilon_0} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{\text{ind}}}{\epsilon_0}$$

$$\sigma_{\text{ind}} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma \tag{26.23}$$

Because $\kappa > 1$, this expression shows that the charge density $\sigma_{\text{ind}}$ induced on the dielectric is less than the charge density $\sigma$ on the plates. For instance, if $\kappa = 3$ we see that the induced charge density is two-thirds the charge density on the plates. If no dielectric is present, then $\kappa = 1$ and $\sigma_{\text{ind}} = 0$ as expected. However, if the dielectric is replaced by an electrical conductor, for which $E = 0$, then Equation 26.22 indicates that $E_0 = E_{\text{ind}}$; this corresponds to $\sigma_{\text{ind}} = \sigma$. That is, the surface charge induced on the conductor is equal in magnitude but opposite in sign to that on the plates, resulting in a net electric field of zero in the conductor (see Fig. 24.16).

We can use the existence of the induced surface charge distributions on the dielectric to explain the result of Example 26.7. As we saw there, the energy of a capacitor not connected to a battery is lowered when a dielectric is inserted between the plates; this means that negative work is done on the dielectric by the external agent inserting the dielectric into the capacitor. This, in turn, implies that a force must be acting on the dielectric that draws it into the capacitor. This force originates from the nonuniform nature of the electric field of the capacitor near its edges, as indicated in Figure 26.27.
The horizontal component of this fringe field acts on the induced charges on the surface of the dielectric, producing a net horizontal force directed into the space between the capacitor plates.

### Example 26.9  Effect of a Metallic Slab

A parallel-plate capacitor has a plate separation \(d\) and plate area \(A\). An uncharged metallic slab of thickness \(a\) is inserted midway between the plates.

**Question (A)** Find the capacitance of the device.

**Solution** We can solve this problem by noting that any charge that appears on one plate of the capacitor must induce a charge of equal magnitude and opposite sign on the near side of the slab, as shown in Figure 26.28a. Consequently, the net charge on the slab remains zero, and the electric field inside the slab is zero. Hence, the capacitor is equivalent to two capacitors in series, each having a plate separation \((d - a)/2\), as shown in Figure 26.28b.

Using Eq. 26.3 and the rule for adding two capacitors in series (Eq. 26.10), we obtain

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{(d - a)/2}} + \frac{1}{\frac{\epsilon_0 A}{(d - a)/2}}
\]

\[
C = \frac{\epsilon_0 A}{d - a}
\]

Note that \(C\) approaches infinity as \(a\) approaches \(d\). Why?

**Question (B)** Show that the capacitance of the original capacitor is unaffected by the insertion of the metallic slab if the slab is infinitesimally thin.

**Solution** In the result for part (A), we let \(a \to 0\):

\[
C = \lim_{a \to 0} \frac{\epsilon_0 A}{d - a} = \frac{\epsilon_0 A}{d}
\]

which is the original capacitance.

**What If?** What if the metallic slab in part (A) is not midway between the plates? How does this affect the capacitance?

**Answer** Let us imagine that the slab in Figure 26.27a is moved upward so that the distance between the upper edge of the slab and the upper plate is \(b\). Then, the distance between the lower edge of the slab and the lower plate is \(d - b - a\). As in part (A), we find the total capacitance of the series combination:

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{b}} + \frac{1}{\frac{\epsilon_0 A}{(d - b - a)}}
\]

\[
C = \frac{\epsilon_0 A}{b} + \frac{d - b - a}{\epsilon_0 A} = \frac{d - a}{\epsilon_0 A} = \frac{\epsilon_0 A}{d - a}
\]

This is the same result as in part (A). It is independent of the value of \(b\), so it does not matter where the slab is located. In Figure 26.28b, when the central structure is moved up or down, the decrease in plate separation of one capacitor is compensated by the increase in plate separation for the other.

**Figure 26.28** (Example 26.9) (a) A parallel-plate capacitor of plate separation \(d\) partially filled with a metallic slab of thickness \(a\). (b) The equivalent circuit of the device in part (a) consists of two capacitors in series, each having a plate separation \((d - a)/2\).

### Example 26.10  A Partially Filled Capacitor

A parallel-plate capacitor with a plate separation \(d\) has a capacitance \(C_0\) in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant \(\kappa\) and thickness \(\frac{1}{3}d\) is inserted between the plates (Fig. 26.29a)?

**Solution** In Example 26.9, we found that we could insert a metallic slab between the plates of a capacitor and consider the combination as two capacitors in series. The resulting capacitance was independent of the location of the slab. Furthermore, if the thickness of the slab approaches zero,
then the capacitance of the system approaches the capacitance when the slab is absent. From this, we conclude that we can insert an infinitesimally thin metallic slab anywhere between the plates of a capacitor without affecting the capacitance. Thus, let us imagine sliding an infinitesimally thin metallic slab along the bottom face of the dielectric shown in Figure 26.29a. We can then consider this system to be the series combination of the two capacitors shown in Figure 26.29b: one having a plate separation $d/3$ and filled with a dielectric, and the other having a plate separation $2d/3$ and air between its plates.

From Equations 26.15 and 26.3, the two capacitances are

$$C_1 = \frac{\kappa \varepsilon_0 A}{d/3} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 A}{2d/3}$$

Using Equation 26.10 for two capacitors combined in series, we have

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa \varepsilon_0 A} + \frac{2d/3}{\varepsilon_0 A}$$

$$\frac{1}{C} = \frac{d}{3 \varepsilon_0 A} \left( \frac{1}{\kappa} + 2 \right) = \frac{d}{3 \varepsilon_0 A} \left( \frac{1 + 2 \kappa}{\kappa} \right)$$

$$C = \left( \frac{3 \kappa}{2 \kappa + 1} \right) \frac{\varepsilon_0 A}{d}$$

Because the capacitance without the dielectric is $C_0 = \varepsilon_0 A/d$, we see that

$$C = \left( \frac{3 \kappa}{2 \kappa + 1} \right) C_0$$

**SUMMARY**

A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance $C$ of any capacitor is the ratio of the charge $Q$ on either conductor to the potential difference $\Delta V$ between them:

$$C = \frac{Q}{\Delta V} \quad \text{(26.1)}$$

The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

The SI unit of capacitance is coulombs per volt, or the farad (F), and $1 \text{ F} = 1 \text{ C/V}$.

Capacitance expressions for various geometries are summarized in Table 26.2.

If two or more capacitors are connected in parallel, then the potential difference is the same across all of them. The equivalent capacitance of a parallel combination of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(26.8)}$$

If two or more capacitors are connected in series, the charge is the same on all of them, and the equivalent capacitance of the series combination is given by
These two equations enable you to simplify many electric circuits by replacing multiple capacitors with a single equivalent capacitance.

Energy is stored in a capacitor because the charging process is equivalent to the transfer of charges from one conductor at a lower electric potential to another conductor at a higher potential. The energy stored in a capacitor with charge $Q$ is

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$  \hspace{1cm} (26.11)

When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a dimensionless factor $\kappa$, called the **dielectric constant**:

$$C = \kappa C_0$$  \hspace{1cm} (26.14)

where $C_0$ is the capacitance in the absence of the dielectric. The increase in capacitance is due to a decrease in the magnitude of the electric field in the presence of the dielectric. The decrease in the magnitude of $E$ arises from an internal electric field produced by aligned dipoles in the dielectric.

The **electric dipole moment** $p$ of an electric dipole has a magnitude

$$p = 2aq$$  \hspace{1cm} (26.16)

The direction of the electric dipole moment vector is from the negative charge toward the positive charge.

The torque acting on an electric dipole in a uniform electric field $E$ is

$$\tau = p \times E$$  \hspace{1cm} (26.18)

The potential energy of the system of an electric dipole in a uniform external electric field $E$ is

$$U = -p \cdot E$$  \hspace{1cm} (26.20)

### Questions

1. The plates of a capacitor are connected to a battery. What happens to the charge on the plates if the connecting wires are removed from the battery? What happens to the charge if the wires are removed from the battery and connected to each other?

2. A farad is a very large unit of capacitance. Calculate the length of one side of a square, air-filled capacitor that has a capacitance of 1 F and a plate separation of 1 m.

3. A pair of capacitors are connected in parallel while an identical pair are connected in series. Which pair would be...
more dangerous to handle after being connected to the same battery? Explain.

4. If you are given three different capacitors \( C_1, C_2, C_3 \), how many different combinations of capacitance can you produce?

5. What advantage might there be in using two identical capacitors in parallel connected in series with another identical parallel pair, rather than using a single capacitor?

6. Is it always possible to reduce a combination of capacitors to one equivalent capacitor with the rules we have developed? Explain.

7. The sum of the charges on both plates of a capacitor is zero. What does a capacitor store?

8. Because the charges on the plates of a parallel-plate capacitor are opposite in sign, they attract each other. Hence, it would take positive work to increase the plate separation. What type of energy in the system changes due to the external work done in this process?

9. Why is it dangerous to touch the terminals of a high-voltage capacitor even after the applied potential difference has been turned off? What can be done to make the capacitor safe to handle after the voltage source has been removed?

10. Explain why the work needed to move a charge \( Q \) through a potential difference \( \Delta V \) is \( W = Q \Delta V \) whereas the energy stored in a charged capacitor is \( U = \frac{1}{2} Q \Delta V \). Where does the \( \frac{1}{2} \) factor come from?

11. If the potential difference across a capacitor is doubled, by what factor does the energy stored change?

12. It is possible to obtain large potential differences by first charging a group of capacitors connected in parallel and then activating a switch arrangement that in effect disconnects the capacitors from the charging source and from each other and reconnects them in a series arrangement. The group of charged capacitors is then discharged in series. What is the maximum potential difference that can be obtained in this manner by using ten capacitors each of 500 \( \mu F \) and a charging source of 800 V?

13. Assume you want to increase the maximum operating voltage of a parallel-plate capacitor. Describe how you can do this for a fixed plate separation.

14. An air-filled capacitor is charged, then disconnected from the power supply, and finally connected to a voltmeter. Explain how and why the potential difference changes when a dielectric is inserted between the plates of the capacitor.

15. Using the polar molecule description of a dielectric, explain how a dielectric affects the electric field inside a capacitor.

16. Explain why a dielectric increases the maximum operating voltage of a capacitor although the physical size of the capacitor does not change.

17. What is the difference between dielectric strength and the dielectric constant?

18. Explain why a water molecule is permanently polarized. What type of molecule has no permanent polarization?

19. If a dielectric-filled capacitor is heated, how will its capacitance change? (Ignore thermal expansion and assume that the dipole orientations are temperature-dependent.)

20. If you were asked to design a capacitor where small size and large capacitance were required, what factors would be important in your design?

---

### PROBLEMS

**Section 26.1 Definition of Capacitance**

1. (a) How much charge is on each plate of a 4.00-\( \mu F \) capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?

2. Two conductors having net charges of + 10.0 \( \mu C \) and − 10.0 \( \mu C \) have a potential difference of 10.0 V between them. (a) Determine the capacitance of the system. (b) What is the potential difference between the two conductors if the charges on each are increased to + 100 \( \mu C \) and − 100 \( \mu C \)?

**Section 26.2 Calculating Capacitance**

3. An isolated charged conducting sphere of radius 12.0 cm creates an electric field of 4.90 \( \times 10^4 \) N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?

4. (a) If a drop of liquid has capacitance 1.00 pF, what is its radius? (b) If another drop has radius 2.00 mm, what is its capacitance? (c) What is the charge on the smaller drop if its potential is 100 V?

5. Two conducting spheres with diameters of 0.400 m and 1.00 m are separated by a distance that is large compared with the diameters. The spheres are connected by a thin wire and are charged to 7.00 \( \mu C \). (a) How is this total charge shared between the spheres? (Ignore any charge on the wire.) (b) What is the potential of the system of spheres when the reference potential is taken to be \( V = 0 \) at \( r = \infty \)?

6. Regarding the Earth and a cloud layer 800 m above the Earth as the “plates” of a capacitor, calculate the
An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

14. A small object of mass m carries a charge q and is suspended by a thread between the vertical plates of a parallel-plate capacitor. The plate separation is d. If the thread makes an angle θ with the vertical, what is the potential difference between the plates?

15. Find the capacitance of the Earth. (Suggestion: The outer conductor of the “spherical capacitor” may be considered as a conducting sphere at infinity where V approaches zero.)

**Section 26.3 Combinations of Capacitors**

16. Two capacitors, \( C_1 = 5.00 \mu\text{F} \) and \( C_2 = 12.0 \mu\text{F} \), are connected in parallel, and the resulting combination is connected to a 9.00-V battery. (a) What is the equivalent capacitance of the combination? What are (b) the potential difference across each capacitor and (c) the charge stored on each capacitor?

17. **What If?** The two capacitors of Problem 16 are now connected in series and to a 9.00-V battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.

18. Evaluate the equivalent capacitance of the configuration shown in Figure P26.18. All the capacitors are identical, and each has capacitance C.

19. Two capacitors when connected in parallel give an equivalent capacitance of 9.00 pF and give an equivalent capacitance of 2.00 pF when connected in series. What is the capacitance of each capacitor?

20. Two capacitors when connected in parallel give an equivalent capacitance of \( C_p \) and an equivalent capacitance of \( C_s \) when connected in series. What is the capacitance of each capacitor?

21. Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if \( \Delta V_{ab} = 15.0 \text{ V} \).
22. Three capacitors are connected to a battery as shown in Figure P26.22. Their capacitances are \( C_1 = 3C \), \( C_2 = C \), and \( C_3 = 5C \). (a) What is the equivalent capacitance of this set of capacitors? (b) State the ranking of the capacitors according to the charge they store, from largest to smallest. (c) Rank the capacitors according to the potential differences across them, from largest to smallest. (d) What If? If \( C_3 \) is increased, what happens to the charge stored by each of the capacitors?

![Figure P26.22](image)

23. Consider the circuit shown in Figure P26.23, where \( C_1 = 6.00 \mu F \), \( C_2 = 3.00 \mu F \), and \( \Delta V = 20.0 \, V \). Capacitor \( C_1 \) is first charged by the closing of switch \( S_1 \). Switch \( S_1 \) is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of \( S_2 \). Calculate the initial charge acquired by \( C_1 \) and the final charge on each capacitor.

![Figure P26.23](image)

24. According to its design specification, the timer circuit delaying the closing of an elevator door is to have a capacitance of 32.0 \( \mu F \) between two points \( A \) and \( B \). (a) When one circuit is being constructed, the inexpensive but durable capacitor installed between these two points is found to have capacitance 34.8 \( \mu F \). To meet the specification, one additional capacitor can be placed between the two points. Should it be in series or in parallel with the 34.8-\( \mu F \) capacitor? What should be its capacitance? (b) What If? The next circuit comes down the assembly line with capacitance 29.8 \( \mu F \) between \( A \) and \( B \). What additional capacitor should be installed in series or in parallel in that circuit, to meet the specification?

25. A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

26. Consider three capacitors \( C_1, C_2, C_3 \), and a battery. If \( C_1 \) is connected to the battery, the charge on \( C_1 \) is 30.8 \( \mu C \). Now \( C_1 \) is disconnected, discharged, and connected in series with \( C_2 \). When the series combination of \( C_2 \) and \( C_1 \) is connected across the battery, the charge on \( C_1 \) is 23.1 \( \mu C \). The circuit is disconnected and the capacitors discharged. Capacitor \( C_3 \), capacitor \( C_1 \), and the battery are connected in series, resulting in a charge on \( C_1 \) of 25.2 \( \mu C \). If, after being disconnected and discharged, \( C_1, C_2, \) and \( C_3 \) are connected in series with one another and with the battery, what is the charge on \( C_1 \)?

27. Find the equivalent capacitance between points \( a \) and \( b \) for the group of capacitors connected as shown in Figure P26.27. Take \( C_1 = 5.00 \mu F, C_2 = 10.0 \mu F, \) and \( C_3 = 2.00 \mu F \).

![Figure P26.27](image)

28. For the network described in the previous problem, if the potential difference between points \( a \) and \( b \) is 60.0 V, what charge is stored on \( C_3 \)?

29. Find the equivalent capacitance between points \( a \) and \( b \) in the combination of capacitors shown in Figure P26.29.

![Figure P26.29](image)

30. Some physical systems possessing capacitance continuously distributed over space can be modeled as an infinite array of discrete circuit elements. Examples are a microwave waveguide and the axon of a nerve cell. To practice analy-
sis of an infinite array, determine the equivalent capacitance \( C \) between terminals \( X \) and \( Y \) of the infinite set of capacitors represented in Figure P26.30. Each capacitor has capacitance \( C_0 \). *Suggestion:* Imagine that the ladder is cut at the line \( AB \), and note that the equivalent capacitance of the infinite section to the right of \( AB \) is also \( C \).

![Figure P26.30](image)

**Section 26.4 Energy Stored in a Charged Capacitor**

31. (a) A 3.00-\( \mu \)F capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

32. The immediate cause of many deaths is ventricular fibrillation, uncoordinated quivering of the heart as opposed to proper beating. An electric shock to the chest can cause momentary paralysis of the heart muscle, after which the heart will sometimes start organized beating again. A *defibrillator* (Fig. 26.14) is a device that applies a strong electric shock to the chest over a time interval of a few milliseconds. The device contains a capacitor of several microfarads, charged to several thousand volts. Electrodes called paddles, about 8 cm across and coated with conducting paste, are held against the chest on both sides of the heart. Their handles are insulated to prevent injury to the operator, who calls, “Clear!” and pushes a button on one paddle to discharge the capacitor through the patient’s chest. Assume that an energy of 300 J is to be delivered from a 30.0-\( \mu \)F capacitor. To what potential difference must it be charged?

33. Two capacitors, \( C_1 = 25.0 \) \( \mu \)F and \( C_2 = 5.00 \) \( \mu \)F, are connected in parallel and charged with a 100-V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) *What If?* What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)? Draw a circuit diagram of this circuit.

34. A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled?

35. As a person moves about in a dry environment, electric charge accumulates on his body. Once it is at high voltage, either positive or negative, the body can discharge via sometimes noticeable sparks and shocks. Consider a human body well separated from ground, with the typical capacitance 150 \( \mu \)F. (a) What charge on the body will produce a potential of 10.0 kV? (b) Sensitive electronic devices can be destroyed by electrostatic discharge from a person. A particular device can be destroyed by a discharge releasing an energy of 250 \( \mu \)J. To what voltage on the body does this correspond?

36. A uniform electric field \( E = 5.00 \times 10^5 \) V/m exists within a certain region. What volume of space contains an energy equal to \( 1.00 \times 10^{-7} \) J? Express your answer in cubic meters and in liters.

37. A parallel-plate capacitor has a charge \( Q \) and plates of area \( A \). What force acts on one plate to attract it toward the other plate? Because the electric field between the plates is \( E = Q / \varepsilon_0 A \), you might think that the force is \( F = QE = Q^2 / \varepsilon_0 A \). This is wrong, because the field \( E \) includes contributions from both plates, and the field created by the positive plate cannot exert any force on the positive plate. Show that the force exerted on each plate is actually \( F = Q^2 / 2 \varepsilon_0 A \). *Suggestion:* Let \( C = \varepsilon_0 A / x \) for an arbitrary plate separation \( x \); then require that the work done in separating the two charged plates be \( W = \int F \, dx \).

The force exerted by one charged plate on another is sometimes used in a machine shop to hold a workpiece stationary.

38. The circuit in Figure P26.38 consists of two identical parallel-plate metal plates connected by identical metal springs to a 100-V battery. With the switch open, the plates are uncharged, are separated by a distance \( d = 8.00 \) mm, and have a capacitance \( C = 2.00 \) \( \mu \)F. When the switch is closed, the distance between the plates decreases by a factor of 0.500. (a) *How much charge collects* on each plate and (b) *what is the spring constant for each spring?* *Suggestion:* Use the result of Problem 37.

![Figure P26.38](image)

39. *Review problem.* A certain storm cloud has a potential of \( 1.00 \times 10^8 \) V relative to a tree. If, during a lightning storm, 50.0 C of charge is transferred through this potential difference and 1.00% of the energy is absorbed by the tree, how much sap in the tree can be boiled away? Model the sap as water initially at 30.0°C. Water has a specific heat of 4186 J/kg°C, a boiling point of 100°C, and a latent heat of vaporization of \( 2.26 \times 10^6 \) J/kg.

40. Two identical parallel-plate capacitors, each with capacitance \( C \), are charged to potential difference \( \Delta V \) and connected in parallel. Then the plate separation in one of the capacitors is doubled. (a) Find the total energy of the system of two capacitors before the plate separation is doubled. (b) Find the potential difference across each capacitor after the plate separation is doubled. (c) Find the
total energy of the system after the plate separation is doubled. (d) Reconcile the difference in the answers to parts (a) and (c) with the law of conservation of energy.

41. Show that the energy associated with a conducting sphere of radius \( R \) and charge \( Q \) surrounded by a vacuum is \( U = k_e Q^2 / 2R \).

42. Consider two conducting spheres with radii \( R_1 \) and \( R_2 \). They are separated by a distance much greater than either radius. A total charge \( Q \) is shared between the spheres, subject to the condition that the electric potential energy of the system has the smallest possible value. The total charge \( Q \) is equal to \( q_1 + q_2 \), where \( q_1 \) represents the charge on the first sphere and \( q_2 \) the charge on the second. Because the spheres are very far apart, you can assume that the charge of each is uniformly distributed over its surface. You may use the result of Problem 41. (a) Determine the values of \( q_1 \) and \( q_2 \) in terms of \( Q, R_1, \) and \( R_2 \). (b) Show that the potential difference between the spheres is zero. (We saw in Chapter 25 that two conductors joined by a conducting wire will be at the same potential in a static situation. This problem illustrates the general principle that static charge on a conductor will distribute itself so that the electric potential energy of the system is a minimum.)

Section 26.5 Capacitors with Dielectrics

43. Determine (a) the capacitance and (b) the maximum potential difference that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 \( \text{cm}^2 \) and plate separation of 0.040 \( \text{mm} \).

44. (a) How much charge can be placed on a capacitor with air between the plates before it breaks down, if the area of each of the plates is 5.00 \( \text{cm}^2 \)? (b) What If? Find the maximum charge if polystyrene is used between the plates instead of air.

45. A commercial capacitor is to be constructed as shown in Figure 26.17a. This particular capacitor is made from two strips of aluminum separated by a strip of paraffin-coated paper. Each strip of foil and paper is 7.00 \( \text{cm} \) wide. The foil is 0.00400 \( \text{mm} \) thick, and the paper is 0.0250 \( \text{mm} \) thick and has a dielectric constant of 3.70. What length should the strips have, if a capacitance of 9.50 \times 10^{-8} \( \text{F} \) is desired before the capacitor is rolled up? (Adding a second strip of paper and rolling the capacitor effectively doubles its capacitance, by allowing charge storage on both sides of each strip of foil.)


47. A parallel-plate capacitor in air has a plate separation of 1.50 \( \text{cm} \) and a plate area of 25.0 \( \text{cm}^2 \). The plates are charged to a potential difference of 250 \( \text{V} \) and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor. Assume the liquid is an insulator.

48. A wafer of titanium dioxide (\( \kappa = 173 \)) of area 1.00 \( \text{cm}^2 \) has a thickness of 0.100 \( \text{mm} \). Aluminum is evaporated on the parallel faces to form a parallel-plate capacitor. (a) Calculate the capacitance. (b) When the capacitor is charged with a 12.0-V battery, what is the magnitude of charge delivered to each plate? (c) For the situation in part (b), what are the free and induced surface charge densities? (d) What is the magnitude of the electric field?

49. Each capacitor in the combination shown in Figure P26.49 has a breakdown voltage of 15.0 \( \text{V} \). What is the breakdown voltage of the combination?

\[ \begin{align*}
20.0 \mu F & \quad 20.0 \mu F \\
10.0 \mu F & \quad 20.0 \mu F
\end{align*} \]

Figure P26.49

Section 26.6 Electric Dipole in an Electric Field

50. A small rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge has coordinates \((-1.20 \text{ mm}, 1.10 \text{ mm})\) and the negative charge is at the point \((1.40 \text{ mm}, -1.30 \text{ mm})\). (a) Find the electric dipole moment of the object. The object is placed in an electric field \( E = (7800 \hat{i} - 4900 \hat{j}) \text{ N/C} \). (b) Find the torque acting on the object. (c) Find the potential energy of the object–field system when the object is in this orientation. (d) If the orientation of the object can change, find the difference between the maximum and minimum potential energies of the system.

51. A small object with electric dipole moment \( \vec{p} \) is placed in a nonuniform electric field \( \vec{E} = E(x) \hat{i} \). That is, the field is in the \( x \) direction and its magnitude depends on the coordinate \( x \). Let \( \theta \) represent the angle between the dipole moment and the \( x \) direction. (a) Prove that the dipole feels a net force

\[ F = \vec{p} \left( \frac{dE}{dx} \right) \cos \theta \]

in the direction toward which the field increases. (b) Consider a spherical balloon centered at the origin, with radius 15.0 cm and carrying charge 2.00 \( \mu \text{C} \). Evaluate \( dE/dx \) at the point (16 cm, 0, 0). Assume a water droplet at this point has an induced dipole moment of 6.301 nC⋅m. Find the force on it.

Section 26.7 An Atomic Description of Dielectrics

52. A detector of radiation called a Geiger tube consists of a closed, hollow, conducting cylinder with a fine wire along its axis. Suppose that the internal diameter of the cylinder is 2.50 cm and that the wire along the axis has a diameter of 0.200 mm. The dielectric strength of the gas between the central wire and the cylinder is 1.20 \times 10^6 \( \text{V/m} \). Calculate the maximum potential difference that can be applied between the wire and the cylinder before breakdown occurs in the gas.
53. The general form of Gauss’s law describes how a charge creates an electric field in a material, as well as in vacuum. It is
\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon} \]
where \( \varepsilon = \varepsilon_0 \) is the permittivity of the material. (a) A sheet with charge \( Q \) uniformly distributed over its area \( A \) is surrounded by a dielectric. Show that the sheet creates a uniform electric field at nearby points, with magnitude \( E = Q/2A\varepsilon \). (b) Two large sheets of area \( A \), carrying opposite charges of equal magnitude \( Q \), are a small distance \( d \) apart. Show that they create uniform electric field in the space between them, with magnitude \( E = Q/A\varepsilon \). (c) Assume that the negative plate is at zero potential. Show that the positive plate is at potential \( Qd/A\varepsilon \). (d) Show that the capacitance of the pair of plates is \( \Lambda\varepsilon/d = \kappa A\varepsilon_0/d \).

Additional Problems

54. For the system of capacitors shown in Figure P26.54, find (a) the equivalent capacitance of the system, (b) the potential across each capacitor, (c) the charge on each capacitor, and (d) the total energy stored by the group.

\[ \begin{align*}
&3.00 \mu F & 6.00 \mu F \\
&2.00 \mu F & 4.00 \mu F \\
&90.0 \text{ V} \\
\end{align*} \]

Figure P26.54

55. Four parallel metal plates \( P_1, P_2, P_3 \), and \( P_4 \), each of area 7.50 cm\(^2\), are separated successively by a distance \( d = 1.19 \text{ mm} \), as shown in Figure P26.55. \( P_1 \) is connected to the negative terminal of a battery, and \( P_2 \) to the positive terminal. The battery maintains a potential difference of 12.0 V. (a) If \( P_3 \) is connected to the negative terminal, what is the capacitance of the three-plate system \( P_1P_2P_3 \)? (b) What is the charge on \( P_2^2 \)? (c) If \( P_4 \) is now connected to the positive terminal of the battery, what is the capacitance of the four-plate system \( P_1P_2P_3P_4 \)? (d) What is the charge on \( P_4^2 \)?

56. One conductor of an overhead electric transmission line is a long aluminum wire 2.40 cm in radius. Suppose that at a particular moment it carries charge per length 1.40 \( \mu \text{C/m} \) and is at potential 345 kV. Find the potential 12.0 m below the wire. Ignore the other conductors of the transmission line and assume the electric field is everywhere purely radial.

57. Two large parallel metal plates are oriented horizontally and separated by a distance \( 3d \). A grounded conducting wire joins them, and initially each plate carries no charge. Now a third identical plate carrying charge \( Q \) is inserted between the two plates, parallel to them and located a distance \( d \) from the upper plate, as in Figure P26.57. (a) What induced charge appears on each of the two original plates? (b) What potential difference appears between the middle plate and each of the other plates? Each plate has area \( A \).

58. A 2.00-nF parallel-plate capacitor is charged to an initial potential difference \( \Delta V = 100 \text{ V} \) and then isolated. The dielectric material between the plates is mica, with a dielectric constant of 5.00. (a) How much work is required to withdraw the mica sheet? (b) What is the potential difference of the capacitor after the mica is withdrawn?

59. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is 3.00 and whose dielectric strength is \( 2.00 \times 10^8 \text{ V/m} \). The desired capacitance is 0.250 \( \mu \text{F} \), and the capacitor must withstand a maximum potential difference of 4000 V. Find the minimum area of the capacitor plates.

60. A 10.0-\( \mu \text{F} \) capacitor has plates with vacuum between them. Each plate carries a charge of magnitude 1000 \( \mu \text{C} \). A particle with charge \( -3.00 \mu \text{C} \) and mass \( 2.00 \times 10^{-16} \text{ kg} \) is fired from the positive plate toward the negative plate with an initial speed of \( 2.00 \times 10^5 \text{ m/s} \). Does it reach the negative plate? If so, find its impact speed. If not, what fraction of the way across the capacitor does it travel?

61. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure P26.61. You may assume that \( \ell \gg d \). (a) Find an expression for the capacitance of the device in terms of the plate area \( A \) and \( d \), \( \kappa_1, \kappa_2 \), and \( \kappa_3 \). (b) Calculate the capacitance using the values \( A = 1.00 \text{ cm}^2 \), \( d = 2.00 \text{ mm} \), \( \kappa_1 = 4.90 \), \( \kappa_2 = 5.60 \), and \( \kappa_3 = 2.10 \).
62. A 10.0-\(\mu\)F capacitor is charged to 15.0 V. It is next connected in series with an uncharged 5.00-\(\mu\)F capacitor. The series combination is finally connected across a 50.0-V battery, as diagrammed in Figure P26.62. Find the new potential differences across the 5-\(\mu\)F and 10-\(\mu\)F capacitors.

\[
\begin{align*}
5.00 \mu F & \quad + \quad 10.0 \mu F \\
\Delta V &= 15.0 V
\end{align*}
\]

50.0 V

Figure P26.62

63. (a) Two spheres have radii \(a\) and \(b\) and their centers are a distance \(d\) apart. Show that the capacitance of this system is

\[
C = \frac{4\pi \varepsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}
\]

provided that \(d\) is large compared with \(a\) and \(b\). (Suggestion: Because the spheres are far apart, assume that the potential of each equals the sum of the potentials due to each sphere, and when calculating those potentials assume that \(V = kQ/r\) applies.) (b) Show that as \(d\) approaches infinity the above result reduces to that of two spherical capacitors in series.

64. A capacitor is constructed from two square plates of sides \(\ell\) and separation \(d\). A material of dielectric constant \(\kappa\) is inserted a distance \(x\) into the capacitor, as shown in Figure P26.64. Assume that \(d\) is much smaller than \(x\). (a) Find the equivalent capacitance of the device. (b) Calculate the energy stored in the capacitor, letting \(\Delta V\) represent the potential difference. (c) Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference \(\Delta V\). Ignore friction. (d) Obtain a numerical value for the force assuming that \(\ell = 5.00\) cm, \(\Delta V = 2000\) V, \(d = 2.00\) mm, and the dielectric is glass (\(\kappa = 4.50\)). (Suggestion: The system can be considered as two capacitors connected in parallel.)

65. A capacitor is constructed from two square plates of sides \(\ell\) and separation \(d\), as suggested in Figure P26.64. You may assume that \(d\) is much less than \(\ell\). The plates carry charges \(+Q_0\) and \(-Q_0\). A block of metal has a width \(\ell\), a length \(\ell\), and a thickness slightly less than \(d\). It is inserted a distance \(x\) into the capacitor. The charges on the plates are not disturbed as the block slides in. In a static situation, a metal prevents an electric field from penetrating inside it. The metal can be thought of as a perfect dielectric, with \(\kappa \to \infty\). (a) Calculate the stored energy as a function of \(x\). (b) Find the direction and magnitude of the force that acts on the metallic block. (c) The area of the advancing front face of the block is essentially equal to \(\ell d\). Considering the force on the block as acting on this face, find the stress (force per area) on it. (d) For comparison, express the energy density in the electric field between the capacitor plates in terms of \(Q_0\), \(\ell\), \(d\), and \(\varepsilon_0\).

66. When considering the energy supply for an automobile, the energy per unit mass of the energy source is an important parameter. Using the following data, compare the energy per unit mass (J/kg) for gasoline, lead–acid batteries, and capacitors. (The ampere A will be introduced in the next chapter as the SI unit of electric current. 1 A = 1 C/s.)

- **Gasoline**: 126 000 Btu/gal; density = 670 kg/m\(^3\).
- **Lead–acid battery**: 12.0 V; 100 Ah; mass = 16.0 kg.
- **Capacitor**: potential difference at full charge = 12.0 V; capacitance = 0.100 F; mass = 0.100 kg.

67. An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10.0-\(\mu\)F capacitor, the potential difference across the combination is 30.0 V. Calculate the unknown capacitance.

68. To repair a power supply for a stereo amplifier, an electronics technician needs a 100-\(\mu\)F capacitor capable of withstanding a potential difference of 90 V between the plates. The only available supply is a box of five 100-\(\mu\)F capacitors, each having a maximum voltage capability of 50 V. Can the technician substitute a combination of these capacitors that has the proper electrical characteristics? If so, what will be the maximum voltage across any of the capacitors used? (Suggestion: The technician may not have to use all the capacitors in the box.)

69. A parallel-plate capacitor of plate separation \(d\) is charged to a potential difference \(\Delta V_0\). A dielectric slab of thickness \(d\) and dielectric constant \(\kappa\) is introduced between the plates while the battery remains connected to the plates. (a) Show that the ratio of energy stored after the dielectric is introduced to the energy stored in the empty capacitor is \(U/U_0 = \kappa\). Give a physical explanation for this increase in stored energy. (b) What happens to the charge on the capacitor? (Note that this situation is not the same as in
Example 26.7, in which the battery was removed from the circuit before the dielectric was introduced.)

70. A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is 2.00 (Fig. P26.70a). When this capacitor is positioned horizontally, the space between the conductors is 7.3 mm. The maximum potential difference capability is attained when the outer conductor is connected to each other. (Suggestion: Consider the symmetry involved.)

![Figure P26.70](image)

71. Capacitors \( C_1 = 6.00 \mu \text{F} \) and \( C_2 = 2.00 \mu \text{F} \) are charged as a parallel combination across a 250-V battery. The capacitors are disconnected from the battery and from each other. They are then connected positive plate to negative plate and negative plate to positive plate. Calculate the resulting charge on each capacitor.

72. Calculate the equivalent capacitance between the points \( a \) and \( b \) in Figure P26.72. Note that this is not a simple series or parallel combination. (Suggestion: Assume a potential difference \( \Delta V \) between points \( a \) and \( b \). Write expressions for \( \Delta V_{ab} \) in terms of the charges and capacitances for the various possible pathways from \( a \) to \( b \), and require conservation of charge for those capacitor plates that are connected to each other.)

![Figure P26.72](image)

73. The inner conductor of a coaxial cable has a radius of 0.800 mm, and the outer conductor’s inside radius is 3.00 mm. The space between the conductors is filled with polyethylene, which has a dielectric constant of 2.30 and a dielectric strength of \( 18.0 \times 10^6 \text{ V/m} \). What is the maximum potential difference that this cable can withstand?

74. You are optimizing coaxial cable design for a major manufacturer. Show that for a given outer conductor radius \( b \), maximum potential difference capability is attained when the radius of the inner conductor is \( a = b/e \) where \( e \) is the base of natural logarithms.

75. Determine the equivalent capacitance of the combination shown in Figure P26.75. (Suggestion: Consider the symmetry involved.)

![Figure P26.75](image)

76. Consider two long, parallel, and oppositely charged wires of radius \( d \) with their centers separated by a distance \( D \). Assuming the charge is distributed uniformly on the surface of each wire, show that the capacitance per unit length of this pair of wires is

\[
\frac{C}{\ell} = \frac{\pi \varepsilon_0}{\ln[(D - d)/d]}
\]

77. Example 26.2 explored a cylindrical capacitor of length \( \ell \) and radii \( a \) and \( b \) of the two conductors. In the What If? section, it was claimed that increasing \( \ell \) by 10% is more effective in terms of increasing the capacitance than increasing \( a \) by 10% if \( b > 2.85a \). Verify this claim mathematically.

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**Answers to Quick Quizzes**

26.1 (d). The capacitance is a property of the physical system and does not vary with applied voltage. According to Equation 26.1, if the voltage is doubled, the charge is doubled.

26.2 (a). When the key is pressed, the plate separation is decreased and the capacitance increases. Capacitance depends only on how a capacitor is constructed and not on the external circuit.

26.3 (a). When connecting capacitors in series, the inverses of the capacitances add, resulting in a smaller overall equivalent capacitance.

26.4 (a). When capacitors are connected in series, the voltages add, for a total of 20 V in this case. If they are combined in parallel, the voltage across the combination is still 10 V.

26.5 (b). For a given voltage, the energy stored in a capacitor is proportional to \( C \): \( U = C(\Delta V)^2/2 \). Thus, you want to maximize the equivalent capacitance. You do this by connecting the three capacitors in parallel, so that the capacitances add.

26.6 (a) \( C \) decreases (Eq. 26.3). (b) \( Q \) stays the same because there is no place for the charge to flow. (c) \( E \) remains constant (see Eq. 24.8 and the paragraph following it). (d) \( \Delta V \) increases because \( \Delta V = Q/C \), \( Q \) is constant (part b), and \( C \) decreases (part a). (e) The energy stored in the capacitor is proportional to both \( Q \) and \( \Delta V \) (Eq.
26.11) and thus increases. The additional energy comes from the work you do in pulling the two plates apart.

26.7 (a) $C$ decreases (Eq. 26.3). (b) $Q$ decreases. The battery supplies a constant potential difference $\Delta V$; thus, charge must flow out of the capacitor if $C = Q/\Delta V$ is to decrease. (c) $E$ decreases because the charge density on the plates decreases. (d) $\Delta V$ remains constant because of the presence of the battery. (e) The energy stored in the capacitor decreases (Eq. 26.11).

26.8 Increase. The dielectric constant of wood (and of all other insulating materials, for that matter) is greater than 1; therefore, the capacitance increases (Eq. 26.14). This increase is sensed by the stud-finder’s special circuitry, which causes an indicator on the device to light up.

26.9 (a) $C$ increases (Eq. 26.14). (b) $Q$ increases. Because the battery maintains a constant $\Delta V$, $Q$ must increase if $C$ increases. (c) $E$ between the plates remains constant because $\Delta V = Ed$ and neither $\Delta V$ nor $d$ changes. The electric field due to the charges on the plates increases because more charge has flowed onto the plates. The induced surface charges on the dielectric create a field that opposes the increase in the field caused by the greater number of charges on the plates (see Section 26.7). (d) The battery maintains a constant $\Delta V$. 